Developing 3D Naiver stoke solver using FVM and GPU programing: Survey

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Abstract—In CFD behavior of fluid is simulated with the help of Naiver stokes equation. Solving NS equation requires high computations. In this paper, we review various methods to solve NS (Naiver-Stokes) equation using the different algorithms but main focus is on solving 3D naiver stokes. Various methods used to solve but FVM method is best for 3D Navier stokes equation. To solve NS equation high computation is required. To speed up the high calculation GPU based parallel programming is useful. In this paper difference between various methods are highlighted. CUDA Framework provides access to huge number of SM which are in GPU. The paper starts by briefly exploring CFD and NS equation then literature survey after that GPU programming model and at last discussed 3D NS equation and methodology to solve.

Keywords—High Performance computing (HPC), Computational Fluid Dynamics(CFD), NS(Navier Stokes),GPU**.**

I. INTRODUCTION

Now a days high computational power computers are used in every filed for example in biology, in erodynamics, chemical industry ,CFD and so many other fields. Computational fluid dynamics is branch of fluid mechanics. It deals with fluid motion. In many filed simulation is required .In CFD many problems related to fluid are solved By numerical computation and by using algorithms. The Naiver stokes equations have great importance in the field of scientific research and Engineering field. Naiver stokes shows the fluid motion. It can used to model ocean current, weather forecasting, automobile industry, biology and also to simulate flow in the pipe. There are different NS equations like 1D NS equation(Burger equation), 2D NS equation which is also called as heat equation, and last is 3D NS equation.it is also differentiated based on steady, unsteady, comp**ressible, incompressible**. NS equation contains various factors like pressure, velocity, temperature. Various methods are used to solve 1D, 2D NS equations. In 3D NS some solutions are provided by the researchers but that solutions are not reliable and time efficient.

To solve NS equations various discretization methods like Finite difference method, finite volume method, finite element method and boundary element method are used. Solving NS equation requires huge computation to faster the computation

Various programming approaches are used like MPI programming, GPU, OpenCL and various other parallel programming techniques are used.

II. LITERATURE SURVEY

In solving this particular partial differential equations selection of appropriate numerical method leads to greater convergent solution so proper numerical scheme is essential. Kutluay et al. [2], have presented explicit finite difference methods for solving these equations. They have discretized the diffusion equation from Hopf-Cole transformation and gave an explicit expression for the exact solution of it. But they used severe stability condition which requires small size of time steps. Discretization scheme used by them was explicit.

The finite difference method (FDM) is simple to Implement, efficient and flexible. It can be used in fixed boundary problems but it is not useful in the complex geometry object. It can not give efficient convergence rate also it can not handle higher dimension object. Finite element method which is introduced by Luo Yan and Xu You-cai[9] is better than finite difference method. Feature of FEM is it can handle complex geometry as compare to FDM. FVM approximation quality is higher than FDM. FVM is easy to implement. Drawback of FVM is ,it gives pollution error as we move on to higher order equations. Doing parallelization of FVM code is little bit difficult.

Link Ji and Jianxin Zhou [10] uses method to solve NS equation know as boundary element method (BEM). This method is used to solve problems in a directional domain. The benefit of BEM is that there is no need to discretize complete region of flow field instead discretization takes place on the surface of the object. Due this it require less computation time because of the smaller equation system. BEM is not suitable for nonlinear fluid flow problem.

Most of CFD applications uses the finite volume method. FVM is introduced by McDonald, MacCormack, and Paullay. Stelian Ion and Anca Veronica Ion [11] used the FVM method to solve Navier stokes equations. FVM is used to discretize the space into small control volumes. In another paper the author in [12] solved NS equation for incompressible flow on unstructured meshes using finite volume method. FVM is useful for complex geometry object.

Kadalbajoo et al. [3], have applied the Crank-Nicolson method to a linearized equation to obtain unconditionally stable implicit scheme for solving Burgers equation. In this scheme, they do not require any restriction over mesh sizes but the solution obtained from this scheme is with error and thus, not precise.

lattice Boltzman method is used in the work to solve two-dimensional incompressible Navier-Stokes equation. Numerical method is implemented in sequential and CUDA program. Results are then compared with the analytical solution that exists. Computational performance is increased using CUDA. Speedup is almost 10 times than that of sequential
program[4].

Li, Desheng et al. [4], have used linear finite difference Scheme for the initial-boundary problem of Rosenau-Burgers equation. The result obtained by this finite difference scheme are with lesser error than crank-Nicolson method.

According to Bell et al. [5], and Hert et al. [6], it is flexible and efficient to apply finite difference method (FDM) to a variety of transient problems with free and fixed boundary domains, but its simplicity cannot provide the sufficient convergence rate. Using FDM we cannot easily handle higher dimensional geometric objects.

K. Pandey et al. [7], applied the Douglas finite difference to the linearized equation to obtain unconditionally stable fourth order accuracy in space and second-order accuracy in time implicit scheme for solving Burgers equation. But this paper only gets fourth-order.

Basdevant et al. [8], have presented the various solution methods like Fourier method, Chebyshev-Tau method, Crank-Nicolson (ABCN), ABCN collocation method and finite difference (FD**)** solution using a 2nd order approximation and stretching transformation, but all these methods are used to solve the Burgers equation with small viscosity parameter and it also leads to the geometric inflexibility of the problem.

The other methods like exponential finite difference method has been used by Bhattacharya [9], and Handschuh et al. [10], in their respective papers. Although in these two papers the exponential difference method is used, the error in both of these methods was not negligible.

In the paper written by Inan et al [11], they have used a new scheme where they first smooth the equation using LU decomposition and later applied the implicit exponential finite difference method and completely implicit exponential finite difference method.

Paper by Irad Yavneh [12], discusses about various types of multigrid methods like V-cycle method, W-cycle method, FMG method. This paper also discussed about the efficiency of these cycles/methods.

III. FASTER PROCESSING USING GPU

In CFD problems achieving high accuracy with less computation time is main goal to solve CFD problem. To speed the complex computation various HPC techniques are used. Message passing interface (MPI) is used for message passing multiprocessing whereas Open MP is based on shared memory programming. These two methods are used to do parallel processing of complex code. Another parallel processing technique is GPU programming. GPU is developed for speedup the Graphic operations like rendering high graphics but recently it has been proved that GPU also improves Performance in complex task.

In literature authore[13] used GPU for real time simulation of 3D smoke. Simulation is based on Stams semi lagrangian scheme. GPU has many core which can parallel. GPU architecture is stream based architecture which is suitable for parallel operations. Latest GPUs provide floating point performance and memory andwidth that are faster than CPU.

GPUs are based on the stream processing architecture that is suitable for compute-intensive parallel tasks. Modern GPUs can provide memory bandwidth and floating-point performances that are orders of magnitude faster than a standard CPU.

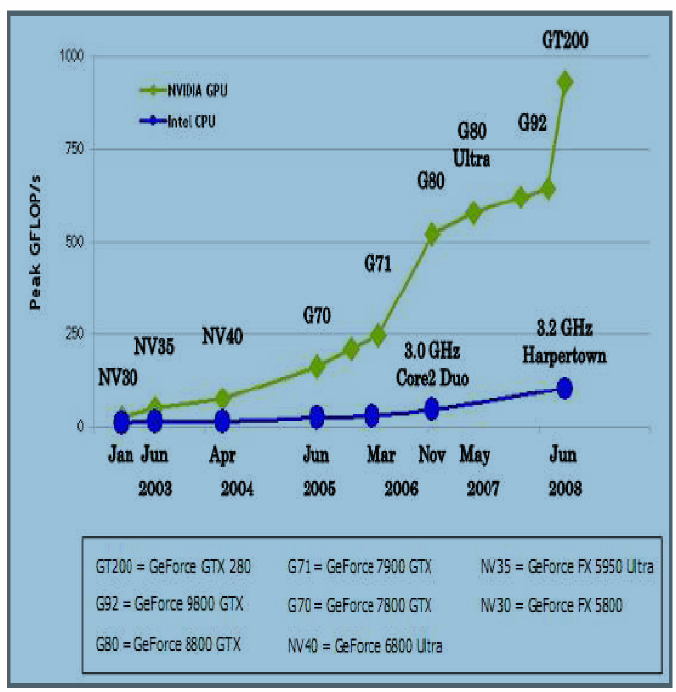

Figure 1. Evolution of performance for CPUs and NVIDIA GPUs (courtesy of NVIDIA).

In the literature[14] author gives the performance diffierence between GPU and CPU. Above figure 1 shows the performance evaluation with respect to year in CPU and GPU.

IV. PROGRAMMING MODEL FOR GPU

GPU are mainly developed for graphic operations but it is Proved that they are gives better performance in high computational job. GPU is of SIMD (Single instruction multiple data) type multiprocessor. Cuda is programming API for GPU programming. In cuda based programming kernel is function which execute on GPU .The task in kernel is performed by multiple processing units using different data. Kernel is called in host function.

GPU runs kernels in order of arrival i.e**.** kernel 1 first and then kernel 2**.** Kernel can call with different parameters. Parameters can be GridDim, BlockDim , ThreadId ,size of shared memory per blocks. New GPU cards have three dimensional grid. Grid can have block in X, Y, Z direction. Block is 3 dimensional in which thread create threads in x, y, z .Blocks within grid have same number of threads. In cuda programming model there are three types of functions i.e. host functions, kernels and device functions.

Host functions execute on CPU while kernels execute task on device. Kernels called from host function. Third type of function is device function which execute on device and called by kernel function. In Cuda there are four types of memories present i.e. host memory which is nothing but CPU RAM accessible to CPU only. While device has constant memory, global memory and shared memory.

V. METHODOLOGY TO SOLVE NAVIER STOKES EQUATION

The Naiver stoke equation is used to solve problems associated to fluid flow. There are various NS equation that is 1D NS equation,2D and 3D NS equations. The simple three dimensional NS equation contains parameters like pressure, velocity and temperature. Three dimensional steady laminar viscous incompressible flows are governed by naiver stokes equations

Continuity equation

$$\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}+\frac{\delta w}{\delta z}=0$$

x - Momentum equation:

$$u\frac{\delta u}{\delta x}+v\frac{\delta u}{\delta y}+w\frac{\delta u}{\delta z}=-\frac{1}{\rho}\frac{\delta p}{\delta x}+\frac{\mu}{\rho}\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}+\frac{\delta^2 u}{\delta z^2}\right)$$

Y - MOMENTUM EQUATION:

$$u\frac{\delta v}{\delta x}+v\frac{\delta v}{\delta y}+w\frac{\delta v}{\delta z}=-\frac{1}{\rho}\frac{\delta p}{\delta x}+\frac{\mu}{\rho}\left(\frac{\delta^2 v}{\delta x^2}+\frac{\delta^2 v}{\delta y^2}+\frac{\delta^2 v}{\delta z^2}\right)$$

z - Momentum equation:

$$u\frac{\delta w}{\delta x}+v\frac{\delta w}{\delta y}+w\frac{\delta w}{\delta z}=-\frac{1}{\rho}\frac{\delta p}{\delta x}+\frac{\mu}{\rho}\left(\frac{\delta^2 w}{\delta x^2}+\frac{\delta^2 w}{\delta y^2}+\frac{\delta^2 w}{\delta z^2}\right)$$

Equitation 1 is the continuity and Eq. 2, 3, 4 are the momentum equations in x, y, z direction respectively. NS is first discretized using Finite volume method and then solved using SIMPLE Algorithm.

VI. CONCLUSION

In this paper based on some literature studies conducted by researchers we presented pro and cons of different numerical schemes for solving NS equation. And also suggest to use the multigrid method to solve NS equation as it provides a good convergent rate at minimum iterative steps. It is hoped that this survey paper will help to stimulate more research and application of Navier-Stokes equation in computational fluid dynamics (CFD).

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