Fuzzy Risks in potato cultivation and its aggregation

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***Abstract-* In this paper an agricultural model with different stages and different risk factors has been formulated using triangular fuzzy number. For aggregation of all fuzzy risks in this model a new method has been proposed to calculate the belongingness of a triangular fuzzy number to the domain of some linguistic variables assigned by a decision maker including some lemmas. Here each linguistic variable is considered as a triangular fuzzy number. An algorithm regarding to this proposed method has been described. It has been shown that the proposed method is better than the conventional center of gravity method. A numerical example has been taken for cultivation of potato in some region of West Bengal in India to calculate an aggregative risk in this system using the proposed method.**

***Keywords-* Aggregative risk, Triangular fuzzy number, Agriculture, Degree of belongingness.**

I INTRODUCTION

The people of major part of the world spend their live depending on cultivation. As a result a farmer always wants to produce a good quality crop to maximize his/her profit. Now-a-day's various techniques of cultivation have been taken in a scientific way. To get the good production a farmer uses different fertilizers for a fertile land. Also some paste killers and different types of chemicals are used to prevent the diseases of the crops. Different hybrid crops are also cultivated hugely. So, a farmer can get a crop in a quick duration. As a result a crop or various crops is cultivated more than one time in a land in a year. But everything may not be profitable to a farmer. High use of these fertilizers, paste killer, chemicals can be harmful to the crops as well as the soil. So, there arises some risk in producing a better quality crops. Also weather condition is an important risk factor which may effect in a good quality product of the crops. So in a cultivation procedure various action is required and various risk are also arises. Due to the complexity of risk factors and uncertainty of the source of risk this is very difficult to analyze. So the fuzzy methods are applied to evaluate the risk of any system. Fuzzy set was first introduced by Zadeh [17]. After that the use of fuzzy set has been applied to large area of mathematics. Zadeh [18] also introduced the concept of linguistic variable and its application to approximate reasoning. There also have some fuzzy risk analysis problems in different areas. Risk analysis using fuzzy set theory is now a very powerful way. Various methods are there to investigate fuzzy risk analysis in different areas. Ranking method of fuzzy number is an important techniques to evaluate risk analysis. Wang and Luo [14] presented a method of area ranking of fuzzy number based on positive negative ideal points. Raj and kumar [11] presented Ranking alternatives with fuzzy weights using maximizing set and minimizing set. Chen et al [2] introduced Fuzzy risk analysis based on ranking generalized fuzzy numbers with different left heights and right heights. Chen and Wang [1] presented the ranking fuzzy number using α cuts, belief feature and signal/noise ratio for risk analysis of a manufacturing system. Chen and Sanguansat [3] introduced a new fuzzy ranking of generalized fuzzy number for risk analysis. Patra and Mondal [10] presented a new ranking method of generalized trapezoidal fuzzy numbers and applied it to evaluate the risk in diabetes problems. Also there are some similarity measure methods to evaluate the risk analysis. Wei and Chen [15] presented a new similarity measure of generalized trapezoidal fuzzy numbers to evaluate a fuzzy risk analysis using linguistic term values. In 2010 Xu et al [16] also introduced a new similarity measure using COG point of two linguistic valued trapezoidal fuzzy numbers and a new arithmetic operator of linguistic values trapezoidal fuzzy numbers. Using linguistic term Kangari and Riggs [6] proposed a method of risk assessment in 1989. Lee et al.[8] presented a method to estimate the human health risk from ground water contamination and to evaluate possible uncertainties associated with the elements. Lee [7] presented a new method of aggregative risk in a software development problem using fuzzy set theory. There also have different papers in agriculture. Tang et al [13] introduced a rice growth and its productivity model. Stilma et al [12] presented Perception of biodiversity in arable production systems in the Netherlands. De Buck et al [4] introduced a mathematical approach to analyze production and environmental risk in arable farming systems. Nuijten [9] presented a paper to enrich the Combining research styles of the natural and social sciences in agricultural research. Haverkort [5] described ecology of potato cropping systems in relation to latitude and altitude.In this paper a new method $FRAM$ has been established to estimate the total risk in an agricultural system taking different risk factors in that system as trapezoidal fuzzy number. It also has been shown that the proposed method give the more better result that existing COG method. As an example, a potato cultivation system has been taken to aggregate the risk in that system. The rest of the paper organized as follows. In section §2 some preliminaries ideas of fuzzy set and its operations has been discussed. In section §3 model formulation and methodology of finding risk of a system has been introduced. Also an algorithm and flowchart regarding to this problem has been given. In section §4 a numerical example of cultivation of potatoes has been given and in section §5 a conclusion has been followed.

II Preliminaries

Fuzzy Set: Fuzzy sets deal with objects that are matter of degree, with all possible grades of truth between yes or no, and the shades of grey between white and black. So a fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. Let F be a collection of objects and x be an element of F, then a fuzzy set $\tilde{A}$ in F is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in F\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership of x in $\tilde{A}$ which maps F to the membership space M which is considered as the closed interval [0,u], where $0 < u \le 1$.

Fuzzy Number: A fuzzy subset $\tilde{A}$ of real number R with membership function $\mu_{\tilde{A}}(x) : R \to [0,1]$ is called a fuzzy number if $\tilde{A}$ is normal and convex, i.e. $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ge \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2)$ for all $x_1, x_2 \in R$

Some Operations in Fuzzy Numbers: Assume that there are two trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ where $0 \le a_1 \le b_1 \le c_1 \le 1$ and $0 \le a_2 \le b_2 \le c_2 \le 1$, then some arithmetic operations between two triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are defined as follows:

(1) The additional operator denoted by $\oplus$ of $\tilde{A}$ and $\tilde{B}$ is defined as

$$\tilde{A} \oplus \tilde{B} = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2)$$

$$= (a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2)$$

(2) The multiplication operator denoted by $\otimes$ of $\tilde{A}$ and $\tilde{B}$ is defined by

$$\tilde{A} \otimes \tilde{B} = (a_1, b_1, c_1) \otimes (a_2, b_2, c_2) = (a_1 a_2, b_1 b_2, c_1 c_2)$$

(3) The divisional operator denoted by % of $\tilde{A}$ and $\tilde{B}$ is defined as

$$\tilde{A} \% \tilde{B} = (a_1, b_1, c_1) \% (a_2, b_2, c_2) = (a_1/c_2, b_1/b_2, c_1/a_2)$$

$$where\, a/b = \begin{cases} a/b, if\ a < b \\ 1,\ otherwise \end{cases}$$

A. COG Method

Let $\tilde{A} = (a, b, c)$ is a triangular fuzzy number. The Center of gravity i.e., COG of this triangular fuzzy number can be evaluated as

$$G_{\tilde{A}} = \frac{1}{3}(a + b + c)$$

In 1996 Lee proposed a aggregative risk model using COG method in software development. To evaluate the membership value of risk function this COG methods has been used. By this method it is seen that the membership of any triangular fuzzy number in the given seven linguistic intervals only on two intervals and on the other intervals they are zero.

III. Risk Model Formulation in an agricultural system and its Methodology (3)

A. Risk Model Formulation

In an agriculture system a crop is cultivated successfully through some stages ($S_i, i = 1,2,3,...,m$) such as soil preparation, sowing/plantation, nourishment and harvesting. For a good production of a crop each and every stage has equal important. But a farmer may give different importancy to different stages form his prior practical experience of cultivation i.e, in general different stages has different weights ($W_i, i = 1,2,3,...,m$) to harvest the crops. Again each stage preparation for a good production depends on the availability of certain amount of different factors. Suppose the stage nourishment will be complete if the different factors such as supply of water, spraying of paste killer, uprootation of weed etc will be of proper amount. But realistically it is not possible to prepare a stage as a 100% complete form for a farmer due to the lack of availability of proper amount of different factors in that stage. As for example, in the nourishment stage the factor "supply of water" is irrigated to the field. Accidently after that there is a heavy rain. So it causes some bad effect on the crop due to excessive water. Therefore there exists some risk for cultivation of a good amount of crop due to the uncertain availability of different factors which are called here risk factors X_{ij}, $i = 1,2,...,m$ and $j = 1,2,...,n_i$ (where the number of risk factors in each stages is n_i). As the availability of X_{ij} are uncertain and these types of uncertainties are normally expressed in linguistic form, hence the risk value can not be measured as a crisp value. So it is appropriate to measure different risk factors in different stages in cultivation as a grade of risk as well as grade of important of risk factors which will be fuzzy number in nature. Generally a farmer may give different important to different risk factors. So according to their importance there exist a weight w_{ij} corresponding to each risk factor. **In such types of problems our main target is to find out to aggregate all grade of risk and severity of harmfulness of all risk factors in the system such that anyone can predict the risk to cultivate a crop in an area.** Now the general structure of such types of problems can be depicted as follows (Table 1):

B. Fuzzy Risk Aggregation Methodology ($FRAM$)

In the proposed agricultural model, there are different grades of risk and severity of harmfulness of the different risk factors which are to be considered as triangular fuzzy numbers in different stages of the cultivation. Let $\tilde{r}_{ij} = (r_{ij1}, r_{ij2}, r_{ij3})$ and $\tilde{s}_{ij} = (s_{ij1}, s_{ij2}, s_{ij3})$ are the grades of risk and severity of harmfulness of j^{th} risk factor in i^{th} stage respectively. Now the product of two fuzzy numbers $\tilde{r}_{ij}$ and $\tilde{s}_{ij}$ is denoted by $\tilde{R}_{ij}$ which is also again a triangular fuzzy number by property. So

$$\tilde{R}_{ij} = \tilde{r}_{ij} \otimes \tilde{s}_{ij} = (a^{ij}, b^{ij}, c^{ij})$$

Now for a large amount of risk factors there will be large amount of triangular fuzzy numbers $\tilde{R}_{ij}$. So it will be difficult in calculation of risk with these different fuzzy numbers. To reduce such large calculations some ξ numbers of specific linguistic variables (Table 2) whose domains belong to $[0,1]$, are considered as triangular fuzzy numbers defined by decision maker from his/her prior experience about the system. To analyze the aggregative risk in the system the degree of belongingness of the triangular fuzzy number $\tilde{R}_{ij}$ is considered on the domain of those linguistic variables which is described as follows (Table 2).

It is considered that the membership functions of the linguistic variables are

$$\mu_k = \begin{cases} 0, & if -\infty < x \le a_k \\ \dfrac{x - a_k}{b_k - a_k}, & if\ a_k < x < b_k \\ 1, & if\ x = b_k \\ \dfrac{c_k - x}{c_k - b_k}, & if\ b_k < x \le c_k \\ 0, & if\ c_k < x < \infty \end{cases}$$

$$a_k \le b_k \le c_k \quad \forall k \in \{1,2,...,\xi\}$$

$$with\ the\ condition\ a_k \le a_{k+1}, b_k \le b_{k+1}, c_k \le c_{k+1} \quad \forall k \in \{1,2,...,\xi - 1\}$$

where three equalities do not hold simultaneously because two triangular fuzzy numbers are distinct. Also

$$a_k \le a_{k+1} \le c_k \quad \forall k \in \{1,2,...,(\xi - 1)\}\ and \quad a_{k+1} \ge b_k\ or\ a_{k+1} \le b_k$$

Now our aim is to find out the belongingness of $\tilde{R}_{ij}$ in the domain of linguistic variables where the membership function of $\tilde{R}_{ij}$ is μ_R^{ij} obtained from equation (5) and presented by bold lines geometrically in the fig 1. Suppose μ_R^{ij} belongs to the domains of p adjacent linguistic variables such as $i^{th}, (i+1)^{th}, ..., (i+p)^{th}$ whose membership functions are μ_i, μ_{i+1},..., μ_{i+p} depicted in fig 1.

As each domain contains μ_R^{ij} then there will be some belongingness of μ_R^{ij} in those linguistic domains. So, in this paper it has been proposed that the belongingness of this $\widetilde{R}_{ij}$ in the domain of k^{th} linguistic variable can be measured by V_k^{ij} as follows

$$V_k^{ij} = \begin{cases} \dfrac{\int_{a^{ij}}^{c^{ij}} x\mu_k \mu_R^{ij} d(\mu_k \mu_R^{ij})}{\int_{a^{ij}}^{c^{ij}} x d(\mu_k \mu_R^{ij})}, \\ \text{if } \widetilde{R}_{ij} \text{ belongs to } k^{th} \text{ linguistic variable} \\ 0, \\ \text{if } \widetilde{R}_{ij} \text{ does not belongs to } k^{th} \text{ linguistic variable} \end{cases} \quad (6)$$

where $k = 1,2,...\xi$ and $V_k^{ij} \geq 0$. Therefore the degree of belongingness of $\widetilde{R}_{ij}$ in the domain of k^{th} linguistic variable, denoted by ζ_k^{ij} can be calculated as $\zeta_k^{ij} = V_k^{ij} / \sum_{r=1}^{\xi} V_r^{ij}$

Lemma 1: The value of ζ_k^{ij} will be zero if $\widetilde{R}_{ij}$ does not belong to the domain of k^{th} linguistic variable.

Proof: If $\widetilde{R}_{ij}$ does not belong to the domain of k^{th} linguistic variable then $V_k^{ij} = 0$ on that domain by equation (6). Hence, $\zeta_k^{ij} = V_k^{ij} / \sum_{r=1}^{\xi} V_r^{ij} = 0$.

Lemma 2: The value of ζ_k^{ij} will be 1 if $\widetilde{R}_{ij}$ belongs to the domain of only k^{th} linguistic variable.

Proof: Let V_r^{ij} is the belongingness of $\widetilde{R}_{ij}$ in the domain of r^{th} linguistic variable. When $\widetilde{R}_{ij}$ only belongs to the domain of k^{th} linguistic variable then

$$V_r^{ij} = 0 \quad \forall r \in \{1,2,...,\xi\} \text{ and } r \neq k.$$

$$i.e., \sum_{r=1}^{\xi} V_r^{ij} = V_k^{ij}$$

$$i.e., V_k^{ij} / \sum_{r=1}^{\xi} V_r^{ij} = 1$$

$$i.e., \zeta_k^{ij} = 1$$

Lemma 3: The value of ζ_k^{ij} always lie between 0 and 1.

Proof: From equation (6) it is seen that $V_k^{ij} \geq 0$.

$$Hence, \zeta_k^{ij} = V_k^{ij} / \sum_{r=1}^{\xi} V_r^{ij} \geq 0$$

$$Again\, V_k^{ij} \leq \sum_{r=1}^{\xi} V_r^{ij}$$

$$so\, V_k^{ij} / \sum_{r=1}^{\xi} V_r^{ij} \leq 1$$

$$i.e., \zeta_k^{ij} \leq 1$$

$$Hence\, 0 \leq \zeta_k^{ij} \leq 1$$

Now for the i^{th} stage of the cultivation the degree of belongingness of $\widetilde{R}_{ij}$ for different risk factors ($j = 1,2,...,n_i$) due to belongingness in the domains of different linguistic variables ($k = 1,2,...,\xi$) can be written in the form of relative matrix (M_i) as follows

$$M_i = \begin{pmatrix} \zeta_1^{i1} & \zeta_2^{i1} & .. & .. & \zeta_\xi^{i1} \\ \zeta_1^{i2} & \zeta_2^{i2} & .. & .. & \zeta_\xi^{i2} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \zeta_1^{in_i} & \zeta_2^{in_i} & .. & .. & \zeta_\xi^{in_i} \end{pmatrix}$$

Using this matrix, the risk of agriculture for different risk factors having the weights $W_{i1}, W_{i2}, ..., W_{in_i}$ in i^{th} stage can be measured as follows

$$R_i = (W_{i1}, W_{i2}, ..., W_{in_i}) * M_i$$
$$= (R_{i1}, R_{i2}, R_{i3}, ..., R_{i\xi})$$

Since there are different weights $W_1, W_2, ..., W_m$ in different m stages of cultivation, therefore the total risk in the cultivation with respect to the domain of ξ linguistic variables can be found as follows

$$R' = (W_1, W_2, ..., W_m) *$$

$$\begin{pmatrix} R_{11} & R_{12} & R_{13} & \dots & \dots & \dots & R_{1\xi} \\ R_{21} & R_{22} & R_{23} & \dots & \dots & \dots & R_{2\xi} \\ .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. \\ R_{m1} & R_{m2} & R_{m3} & \dots & \dots & \dots & R_{m\xi} \end{pmatrix}$$
$$= (R_1^{'}, R_2^{'}, R_3^{'}, ..., R_m^{'})$$

where $\sum_{i=1}^{m} R_i^{'} = 1$.

If G_i be the center of gravity for the domain of i^{th} linguistic variable then the total aggregative risk R of the system is evaluated by

$$R = \sum_{i=1}^{m} G_i \times R_i^{'}.$$

Lemma 4: The value of the aggregative risk R always lie between 0 and 1.

Proof: It is seen that $\sum_{i=1}^{m} R_i^{'} = 1$. Also G_i is the COG of the domain of the i^{th} linguistic variable. As the domain of the linguistic variables lie between 0 and 1, then

$$0 \le G_i \le 1$$
$$i.e., 0 \times \sum_{i=1}^{m} R_i^{'} \le \sum_{i=1}^{m} G_i \times R_i^{'} \le 1 \times \sum_{i=1}^{m} R_i^{'}.$$
$$i.e., 0 \le \sum_{i=1}^{m} G_i \times R_i^{'} \le \sum_{i=1}^{m} R_i^{'}.$$
$$i.e., 0 \le R \le 1.$$

Now the above method to aggregate the fuzzy risks due to different factors in different stages in a cultivation can be written stepwise in the following $FRAM$ -Algorithm:

B.1 $FRAM$ -Algorithm

Step 1: determine the number of stages (S_i, $i = 1,2,...,m$) in a cultivation.

Step 2: consider the weight W_i for stage S_i ($i = 1,2,...,m$).

Step 3: determine the risk factors X_{ij} for each stages.

Step 4: consider the weight W_{ij} for each risk factor X_{ij}.

Step 5: find out the grade of fuzzy risk $\tilde{r}_{ij}$ and grade of fuzzy importance $\tilde{s}_{ij}$ of X_{ij} which are taken as a triangular fuzzy number in this paper.

Step 6: Calculate $\tilde{R}_{ij} = \tilde{r}_{ij} \otimes \tilde{s}_{ij}$ for each risk factor.

Step 7: Calculate the degree of belongingness ζ_k^{ij} of $\tilde{R}_{ij}$ in each domain of the linguistic variables considering the table 2.

Step 8: Find out the relative matrix M_i

Step 9: Calculate the value R_i as $R_i = (R_{i1}, R_{i2}, R_{i3}, ..., R_{i\xi})$

Step 10: Calculate $R^{'}$ as $R^{'} = (R_1^{'}, R_2^{'}, R_3^{'}, ..., R_\xi^{'})$

Step 11: Calculate the center of gravity G_i of each domain of linguistic variables

Step 12: Calculate the aggregative risk R of the whole system using the formula

$$R = \sum_{i=1}^{\xi} G_i \times R_i^{'}$$

A flowchart regarding to the above algorithm is given in fig 2:

Lemma 5: The proposed $FRAM$ is better than the COG method

Proof: In this paper the relative matrix M_i can be calculated using equation (6).It can also be calculated by COG method [Lee (1996)]. But our proposed method gives better performance than COG method which is illustrated using fig 3. Now we consider a triangular fuzzy number $\tilde{A} = (a, b, c)$ and we have to find out the belongingness of $\tilde{A}$ in the domains of 7 linguistic variables in Table 4. When centroid method is used to determine such belongingness of $\tilde{A}$ then only two domains $\tilde{V}_2$ and $\tilde{V}_3$ have contribution in belongingness of $\tilde{A}$.

But from the figure it is seen that $\tilde{A}$ belongs to the five domains of five linguistic variables including $\tilde{V}_2$ and $\tilde{V}_3$. So this method can not determine belongingness of $\tilde{A}$ exactly. But by our proposed method each of five domains has proper contribution to calculate the belongingness of $\tilde{A}$ in the domains of seven linguistic variables. there is some lacuna in finding the belongingness of the fuzzy number $\tilde{A}$ on those domains of the linguistic variables. Hence from this illustration it is obvious that our proposed method- $FRAM$ is better than the existing COG method.

IV Numerical Example

Potato is a main crop which is cultivated with huge amount in different states in India specially North East and North West part of India. Each farmer will be very careful to follow the different stages in cultivation such as (i) preparing soil, (ii) plantation of crop and its nourishment and (iii) harvesting and storage in such a way that a good amount of production will be available. But there are some risk factors in different stages in cultivation. Before cultivation of potato it is necessary to know for a farmer what amount of risk exists in a certain region due to different risk factors. As a case study we have considered potato cultivation in West Bengal in India. From the agricultural news bulletin "Unnata prathay alur chash" of potato in West Bengal Government in India it is known that there are different risk factors in different stages of cultivation which are as follows:

(i) For the first stage i.e., preparation of soil:

(a) Rate of PH in soil: For a good production of potato the normal range of PH is $6.5-7.5$. If the soil becomes acidic i.e., the rate of PH descends under 6.5 then the risk arise. To neutralize the acidity of the soil generally Calcium Hydroxide are used to mix with the soil.

(b) Rate of EC in Soil: If the rate of EC of the soil is below 0.5% then there is no risk. But if the percentage grows risk also increase.

(c) Injection of Cow dung: Cow dung is most useful manure in cultivation. For well manured soil generally $10-12$ ton/hector cow dung is employed. In stead of it there will be some risk in cultivation.

(d) Injection of Calcium Hydroxide: The acidity of soil is destroyed mainly injecting Calcium Hydroxide. Generally 1 ton/hector Calcium Hydroxide are injected in acidic soil. But excessive use of it may have bad effect for later cultivation.

(ii) For the second stage i.e., plantation and nourishment of potato:

(a) After the first rowing, water supply is very needful. Normally in the gap of $3-4$ days water has to be supply. If the supply of water is not taken more than 4 days interval then risk is gradually increased.

(b) After the second rowing, water has to be supply in the time interval of $7-10$ days.

(c) Uprootation of weeds: Weeds are uprooted generally through two steps (i) manually and (ii) Spraying of chemicals. It is better to remove the weeds manually. But if the chemicals are used to uprooted the weed then it caused the damage of soil which is a risk for the crops also for later cultivation.

(d) Disease control: Disease control is very necessary in the cultivation of potato. Different types of diseases appears to the potato. Different types of chemicals are use to prevent these disease which causes risk in crops and soil.

(i) For the third stage i.e., harvesting and storage:

(a) Harvesting of the potatoes in proper time is essential. Harvesting has to be complete in the dry weather condition. If rain of fog continue at the harvesting time that causes risk in damaging the potatoes.

(b) storage in a cold place is very important otherwise potato will be deteriorate and that will causes risk.

In a case study at some villages such as Chandrokona, Gopinathpur and Palashchabri in Paschim Medinipur, West Bengal, India it is informed by 100 farmers on the basis of interview that in the stage of preparation of soil they give 34% importance to PH of soil, 28% to EC of soil, 21% to injection of cow dunk and 17% to injection of calcium hydroxide to the field averagely. After that in the stage of plantation and nourishment they give approximately same importance to first rowing, second rowing, weed control and disease control which are about 25%, 24%, 25% and 26% averagely. Also in the last stage such as harvesting and storage they give 57% importance to harvest the crop from the field and then 43% to storage averagely. Now it is asked to all of those farmers that how much risk will be there if they take all the necessary actions in cultivation in a field where the PH level of the soil is 6.05 and EC of the soil is 0.52%. From their description an average grade of risk of the different risk factors are as follows: The grade of risk of the soil is about 0.5 and severity of harmfulness is about 0.9 for PH level of the soil. For that EC level the grade of risk and severity of harmfulness will be 0.2 and 0.8 respectively. If $10-12$ ton/hector cow dunk is injected to the field there may be some risk in cultivation which is about 0.15 and severity of harmfulness is about 0.7. If calcium hydroxide is injected 1 ton/hector to neutralize the acidic soil there may be some risk about 0.2 and severity of harmfulness may be about 0.65. Taking the care about the water supply after first rowing in the interval of $3-4$ days and after second rowing in the interval of $6-7$ days the grade of risk is about 0.18 and about 0.5 respectively. Also the severity of harmfulness is about 0.85 and 0.82 respectively. If the control of weed is taken by spraying the prometin to the field, there will be a grade of risk about 0.5 and severity of harmfulness will be about 0.9. Also if the necessary chemicals are used to disease control, there will be the grade of risk about 0.18 and severity of harmfulness will be about 0.9. In the last stage there is very low risk in cultivation. There is a grade of risk about 0.12 in harvesting time and about 0.2 in storage with severity of harmfulness is about 0.7 and 0.65 respectively. It is also known that the above risk values may be changed slightly due to different weather conditions. So from the above information it is clear that all risk values are of fuzzy nature. Therefore treating the above risk factors and severity of harmfulness as a triangular fuzzy number the following input table (Table 3) has been constructed to find the aggregative risk in cultivation of potato in a certain region.

From this case study it is also known that different farmers gives different importance to the different stages from their prior experience of cultivation in potato. So, due to risk factors there will be different risk in cultivation. Now the purpose of the risk study in this paper is two-fold: (i) to find the risk in the system by aggregating all fuzzy risk factors and (ii) to analyze the weights i.e., importance in the stages in such a way that risk is minimum. Now, suppose that the weights variables are W_1, W_2 and W_3 to the stages S_1, S_2 and S_3 respectively.

Now to calculate the aggregative risk of the said system, in this paper seven linguistic variables has been considered as in Table 4 according to section §3.2.

Using above input parameters and linguistic variables for said problem, the relative matrix can be calculated according to the method $FRAM$ described in section §3.2

$$M_1 = \begin{pmatrix} 0.000 & 0.017 & 0.294 & 0.647 & 0.042 & 0.000 & 0.000 \\ 0.146 & 0.717 & 0.137 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.344 & 0.637 & 0.019 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.251 & 0.695 & 0.054 & 0.000 & 0.000 & 0.000 & 0.000 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0.1359 & 0.7913 & 0.0728 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0147 & 0.4989 & 0.4685 & 0.0178 & 0.0000 & 0.0000 \\ 0.0000 & 0.0112 & 0.3637 & 0.5702 & 0.0549 & 0.0000 & 0.0000 \\ 0.0910 & 0.7923 & 0.1177 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{pmatrix}$$

$$M_3 = \begin{pmatrix} 0.4799 & 0.5201 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2327 & 0.6498 & 0.1176 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{pmatrix}$$

Using these relative matrices (M_1, M_2 and M_3), the risks for different risk factors in different stages are calculated as

$$R_1 = (0.1560, 0.4585, 0.1514, 0.2198, 0.0143, 0.0000, 0.0000)$$
$$R_2 = (0.0576, 0.4102, 0.2595, 0.2550, 0.0180, 0.0000, 0.0000)$$
$$R_3 = (0.3736, 0.5758, 0.0506, 0.0000, 0.0000, 0.0000, 0.0000)$$

Now with respect to seven linguistic variables and different weights (W_1, W_2 and W_3) the total risk can be calculated as

$$R' = (0.1560W_1 + 0.0576W_2 + 0.3736W_3, 0.4585W_1 + 0.4102W_2 + 0.5758W_3, 0.1514W_1 + 0.2595W_2 + 0.0506W_3, 0.2198W_1 + 0.2550W_2, 0.0143W_1 + 0.0180W_2, 0.0000, 0.0000)$$

Finally using those risk values in the domain of different linguistic variables, the total aggregative risk of the whole system is calculated as

$$R = 0.255W_1 + 0.2976W_2 + 0.1336W_3$$
$$subject\ to$$
$$W_1 + W_2 + W_3 = 1$$

Now for different values of W_1, W_2 and W_3 assigned by farmers to different stages in potato cultivation, different risk values can be found.

However farmers usually give a low weight to the third stage of the cultivation i.e., in the harvesting time and give more importance in the first two stages. For this reason giving a constant small weight such as $W_3 = 0.1$ to the third stage, the weights W_1 and W_2 are varied to get sensitive analysis by fig 4.

Now from above figure it is seen that if W_1 is increased and W_2 is decreased, then the risk values are also decreased. On the other hand if W_1 is decreased and W_2 is increased, then the risk values are also increased. Hence it is concluded that a farmer has to be given more importance in the first stage i.e., in the soil preparation stage than the second stage i.e, in the plantation and nourishment stage. Realistically both the stages have to given approximately same weight otherwise the grade of risk value will be increased and that will cause higher risk. So, from this discussion it is clear that first two stages have to given approximately same weight to minimize the risk factor but first stage has to given more weight than second stage. For this above problem if the values of W_1, W_2 and W_3 are considered as $0.5, 0.4$ and 0.1 respectively as for example then the total risk of the system will be 0.2104.

V Conclusion

In this paper a new method has been established to aggregate the fuzzy risks using triangular fuzzy number in an agriculture system. An algorithm of that method has been proposed and some lemmas associated with the method have been proved. It has been also shown that the proposed method gives the better performance than existing COG method. Finally this method has been applied to calculate the total risk in the potato cultivation and it has been shown that a farmer has to be given more importance in the first stage i.e., in the soil preparation stage than the second stage i.e, in the plantation and nourishment stage. Realistically both the stages have to given approximately same weight otherwise the grade of risk value will be increased and that will cause higher risk. The proposed model can be applied to estimate the risk for such kinds of systems.

VI Figures and Tables

Table-1: Tabular representation of agricultural risk model

Stages	Weights of the stages	Risk items	Weights of the risk items	Grade of risk	severity of harmfulness
S_1	W	X_{11}	w_{11}	$\tilde{r}_{11}$	$\tilde{s}_{11}$
		X_{12}	w_{12}	$\tilde{r}_{12}$	$\tilde{s}_{12}$
		X_{13}	w_{13}	$\tilde{r}_{13}$	$\tilde{s}_{13}$
		...	...	...	...
		...	...	...	...
		X_{1n_1}	w_{1n_1}	$\tilde{r}_{1n_1}$	$\tilde{s}_{1n_1}$
S_2	W	X_{21}	w_{21}	$\tilde{r}_{21}$	$\tilde{s}_{21}$
		X_{22}	w_{22}	$\tilde{r}_{22}$	$\tilde{s}_{22}$
		X_{23}	w_{23}	$\tilde{r}_{23}$	$\tilde{s}_{23}$
		...	...	...	...
		...	...	...	...
		X_{2n_2}	w_{2n_2}	$\tilde{r}_{2n_2}$	$\tilde{s}_{2n_2}$
...	...	...	...	...	...
...	...	...	...	...	...
S_m	W	X_{m1}	w_{m1}	$\tilde{r}_{m1}$	$\tilde{s}_{m1}$
		X_{m2}	w_{m2}	$\tilde{r}_{m2}$	$\tilde{s}_{m2}$
		X_{m3}	w_{m3}	$\tilde{r}_{m3}$	$\tilde{s}_{m3}$
		...	...	...	...
		...	...	...	...
		X_{mn_m}	w_{mn_m}	$\tilde{r}_{mn_m}$	$\tilde{s}_{mn_m}$

Table-2 : General table of Linguistic variables and its domains

linguistic variable	Triangular fuzzy number
very low	(a_1, b_1, c_1)
low	(a_2, b_2, c_2)
...	...
...	...
high	$(a_{\xi-1}, b_{\xi-1}, c_{\xi-1})$
very-high	(a_ξ, b_ξ, c_ξ)

Table - 3 : Different input parameters in potato cultivation.

Stages	Weights of the stages	Risk items	Weights of the risk items	Grade of risk	severity of harmfulness
S_1 : preparation of soil	W_1	PH of soil	0.34	(0.35,0.5,0.6)	(0.8,0.9,0.95)
		EC of soil	0.28	(0.1,0.2,0.3)	(0.7,0.8,0.9)
		inject of cow dung	0.21	(0.1,0.15,0.25)	(0.6,0.7,0.8)
		inject of $Ca(OH)_2$	0.17	(0.1,0.2,0.3)	(0.55,0.65,0.75)
S_2 : plantation and nourishment	W_2	Water supply after 1^{st} rowing	0.25	(0.12,0.18,0.25)	(0.8,0.85,0.9)
		Water supply after 2^{nd} rowing	0.24	(0.4,0.5,0.6)	(0.75,0.82,0.9)
		Weed control	0.25	(0.35,0.5,0.6)	(0.85,0.9,0.95)
		Disease control	0.26	(0.12,0.18,0.25)	(0.85,0.9,0.95)
S_3 : Harvesting and storage	W_3	Harvesting	0.57	(0.05,0.12,0.2)	(0.6,0.7,0.8)
		Storage	0.43	(0.08,0.2,0.35)	(0.5,0.65,0.8)

Table- 4 : Linguistic variables and its domains

linguistic variable	Triangular fuzzy number (Domain)
very low	(0.0,0.0,0.1667)
low	(0.0,0.1667,0.3333)
fairly-low	(0.1667,0.3333,0.5)
medium	(0.3333,0.5,0.6667)
fairly-high	(0.5,0.6667,0.8333)
high	(0.6667,0.8333,1.0)
very-high	(0.8333,1.0,1.0)

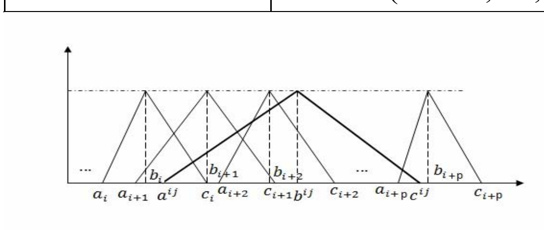

fig 1: representation of belongingness of a triangular fuzzy number $\widetilde{R}_{ij}$

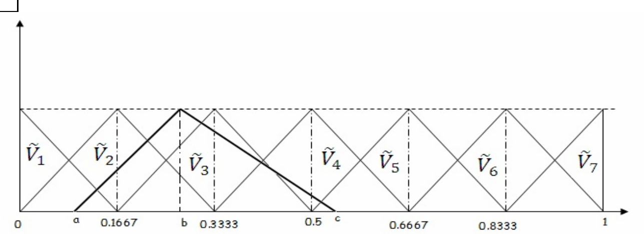

fig 3: representation of linguistic number and a triangular fuzzy number $\widetilde{A} = (a,b,c)$

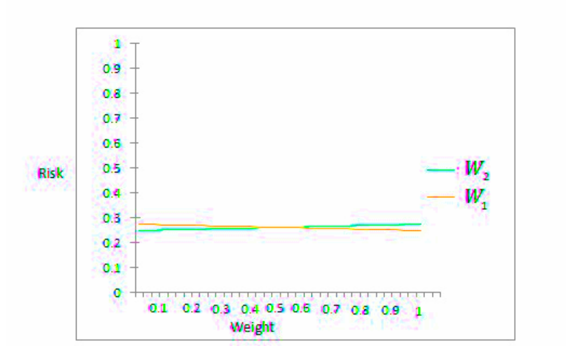

fig 4: representation of risk value with different weights

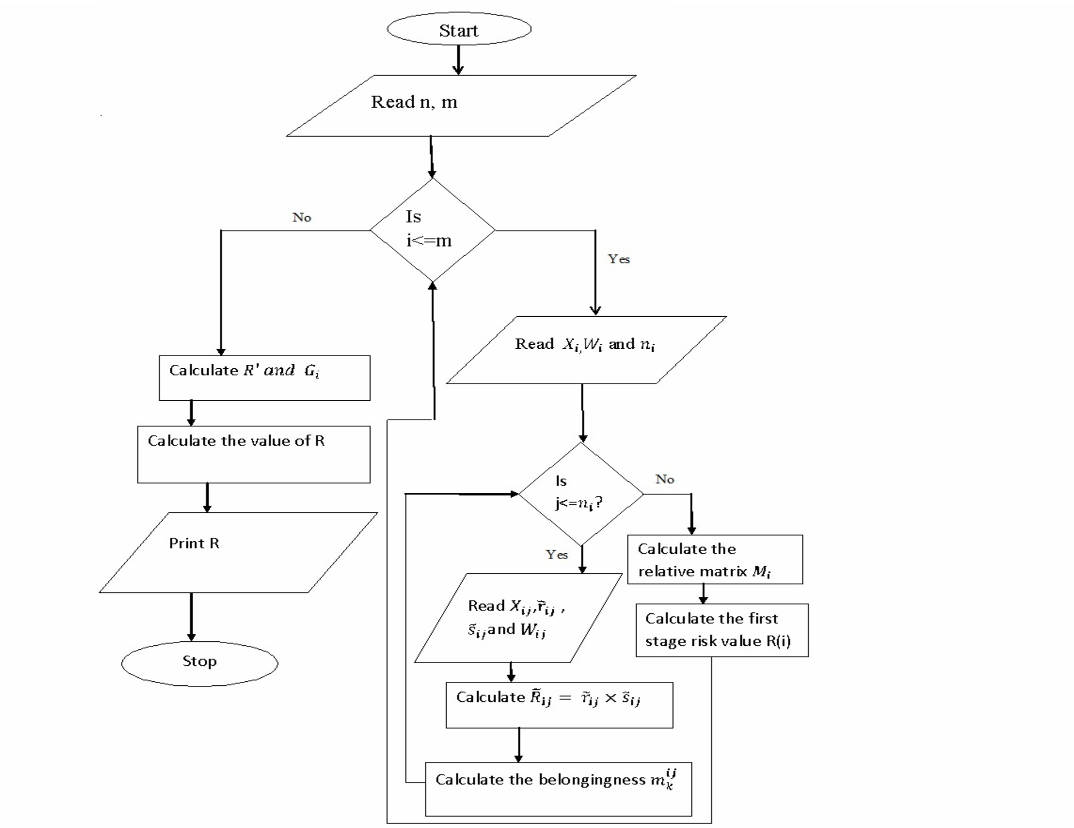

fig 2: Flowchart of the above algorithm

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