

ANALYSIS BY THE BRIDGE DENEUBOURG AXIOMATIC FREQUENTIST DEFINITIONS AND PROBABILITY

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Abstract-In this paper, Deneubourg experiment under two different scenarios, the first branches are considered of equal length and the second length is analyzed differently. Axiomatic (set theory) and frequentist (Monte Carlo simulations): the analysis of the experiment from two definitions of probability is performed. Finally, the code in Matlab with the Monte Carlo simulations were performed are shown.

Keywords-Markov, Monte Carlo, pheromone, Bayes, Chapman-Kolgomorov.

I. INTRODUCTION

An interesting feature of the behavior of ant colonies is how they can find the shortest paths between the nest and food. Along the way they, deposit a substance called pheromone which all can smell, this trail allows the ants back to their nest from the food (Cordon).

Through a simple experiment Deneubourg (Bonabeau 1999) showed that a particular type of ants based selection of the path from the food source to the nest in the accumulation of pheromone in trail. In this experiment, the food source is separated from the nest by a bridge with two branches of equal length and B (Figure 1)no pheromone deposited on the branches, so that each branch has the same probability of being selected. However, random fluctuations determined after a while that most ants select from the branches on the other, say the branch A. Due that ants deposit pheromone as they walk, a greater number of ants on branch A,represents an accumulation of pheromone, thereby encouraging more ants choose the branch A over B.

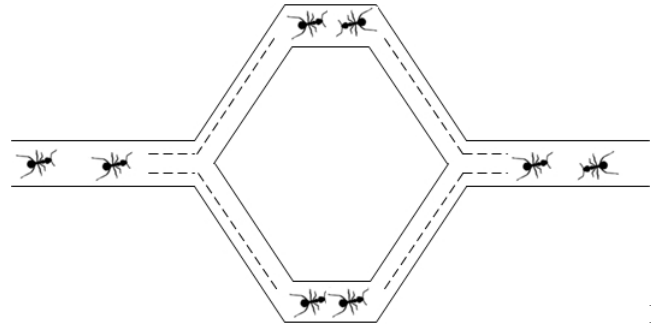


Figure 1. Bridge Deneubourg

Fig

Deneubourg (Bonabeau 1999) developed a probability model of this phenomenon, which largely coincided with experimental observations. In this model, the probability of choosing a certain branch depends on the total number of ants that have used that branch. Thus, A_i and B_i are the number of ants that have used the branches A and B after i ants have used the ant bridge, the probability $P_{A(i+1)}$ ant chooses the branch A is:

$$P_A = \frac{(k + A_i)^n}{(k + A_i)^n + (k + B_i)^n} \quad (1)$$

The parameter n determines the degree of nonlinearity of the selection function. If n is large, and the branch A has slightly more pheromone than branch B, then the ant $(i + 1)$ will have a high probability of choosing the branch A. The parameter k quantifies the attractiveness of non-selected branch, the larger k compared with A_i and B_i , the choice will stop being random and branches will have the same probability of being selected for the $(i + 1)$ ant.

II. METHODOLOGY

Markov model

The model proposed by Deneubourg is a random Markov process (Torres), under which the probability that the ant $(i+1)$ or decide not to go for the branch A depends only on the previous state. So you can define the transition probability:

$$f(z_{i+1} = z_i + 1 | z_i) = \frac{(k_A + z_i)^n}{(k_A + z_i)^n + (k_B + (i - z_i))^n} \quad (2)$$

$$f(z_{i+1} = z_i | z_i) = \frac{(k_B + (i - z_i))^n}{(k_A + z_i)^n + (k_B + (i - z_i))^n}$$

Where z_i is a random variable that takes values in the integer set $\{0, 1, \dots, i\}$ and z_{i+1} in the whole set $\{0, 1, \dots, i, i+1\}$. The basic step in this case is given by the probability of the first ant crossing the bridge,

$$f(z_1 = 1) = \frac{(k_A)^n}{(k_A)^n + (k_B)^n}$$

$$f(z_1 = 0) = \frac{(k_B)^n}{(k_A)^n + (k_B)^n} \tag{3}$$

Note that if $K_A = K_B$ (the branches are of equal length), $f(z_1 = 1) = f(z_1 = 0) = 0.5$, that is, the first ant will have the same probability of picking any branch. Furthermore, according to the properties of a Markov stochastic process, the equations must be met as Kolmogorov Chapman and Bayes (towers), that is,

$$f(z_1, z_3) = f(z_1) \sum_{z_2=0}^2 f(z_2 | z_1) f(z_3 | z_2) \tag{4}$$

$$f(z_{i+1}) = \sum_{z_i=0}^i f(z_{i+1} | z_i) f(z_i) \tag{5}$$

Finally, the joint probability distribution (Quevedo, 2008) of the z_i and z_{i+1} random variables is given by,

Experiment with 3 ants

Then Deneubourg experiment for the case where only three ants are placed on the bridge is discussed. To assist in the analysis, the state transition diagram shown in Figure 2 was constructed. In this diagram, the weight on each branch represents the transition probabilities $f(z_1 | z_2)$ y $f(z_2 | z_3)$, the values on the right of each state, $f(z_1)$ and the joint probabilities $f(z_1, z_2)$, $f(z_2, z_3)$. Calculations were performed with equations (2), (3) and (6)

Using equation (4) and the data in Figure 2a, is the joint probability $f(z_1, z_3)$,

$$f(z_1, z_3) = \begin{matrix} & & & 0 & 1 & 2 & 3 \\ & & & 0 & 0.43 & 0.04 & 0.03 & 0.04 & 0 & 0.43 \\ & & 1 & 0 & 0 & 0.03 & 0 & 0.04 & 0 & 0.43 \end{matrix} \tag{7}$$

Suppose now that the branch A is the winner, which is to say that $z_3 \geq 2$. To analyze what happens in each of the states, from equation (6) the conditional probabilities are defined,

$$f(z_3 | z_3 \geq 2) = \frac{f(z_3, z_3 \geq 2)}{f(z_3 \geq 2)} \tag{8}$$

$$f(z_2 | z_3 \geq 2) = \frac{f(z_2, z_3 \geq 2)}{f(z_3 \geq 2)} \tag{9}$$

$$f(z_1 | z_3 \geq 2) = \frac{f(z_1, z_3 \geq 2)}{f(z_3 \geq 2)} \tag{10}$$

Replacing the data in Figure 2 and the equation (7) into equations (8) - (10), the percentage of passengers (ants) is obtained in the branch A when, it is known that this branch is the winner. The result is shown in the last column of Table 1

Table 1. Percentage of passengers on the winning branch.

z_3	0	1	2	3	Σ	Percentage (%)
$f(z_3 z_3 \geq 2)$	0	0	0.14	0.86		
$z_3 f(z_3 z_3 \geq 2)$	0	0	0.28	2.58	2.86	95.3

z_2	0	1	2	Σ
$f(z_2 z_3 \geq 2)$	0	0.1	0.9	
$z_2 f(z_2 z_3 \geq 2)$		0.1	1.90	1.90
				95

z_1	0	1	Σ
$f(z_1)$	0.06	0.94	
$z_1 f(z_1)$	0	0.94	0.94
			94

Monte Carlo analysis

For statistical modeling of the Deneubourg experiment, the Monte Carlo analysis (Sobol, 1976) is performed. You start by generating random variables with transition probability given in (2) by the biased roulette game (Ross, 2001), ie,

$$z_{i-1} = \begin{cases} z_i + 1 & \text{if } \frac{(k_A + z_i)^n}{(k_A + z_i)^n + (k_B + (i - z_i))^n} \geq \text{Random}(0, 1) \\ z_i & \text{if } \frac{(k_B + (i - z_i))^n}{(k_A + z_i)^n + (k_B + (i - z_i))^n} < \text{Random}(0, 1) \end{cases} \tag{11}$$

Where **Random (0,1)** is a random variable with uniform distribution on the continuous interval (0,1). Matlab pseudocode shown in Table 2.

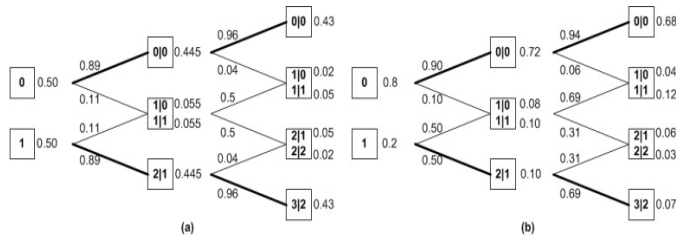


Figure 2. State transition diagram 3 ants (a) $K_A = K_B = 1$ and $n = 3$, (b) $K_A = 1, K_B = 2$ and $n = 2$.

In Figure 2a, note that there is an equal probability that both branches are selected, this is because $K_A = K_B$ (both branches are considered of equal length), while in Figure 2b the branch B (branch less length) is more likely to be selected by ants.

Table 2. Matlab function that generates the random variable z_{i+1} according to (11)

```
function z = Deneubourg(z,KA,KB,n,i)
%Entradas: z (number of ants after i ants are on the
bridge)
%          KA (degree of attraction of branch A)
%          KB (degree of attraction of branch B)
%          n (non-linearity of the selection of the branch
B)
%          i (number of ants that have used the bridge)
%Salida: z (number of ants after i+1 ants are on the
bridge)
fz = ((kA+z)^n/((kA+z)^n+(kB+(i-z))^n);
if (fz>= rand)
z=z+1;
end
```

So, with the function in Table 2 a program in Matlab was designed to conduct the Monte Carlo analysis that obtains the frequency histogram of $f(z)$ and the percentage of passengers in the winning branch. The pseudocode is shown in Table 3.

Table 3. Monte Carlo analysis

```
%Entradas: m (number of experiments)
%          i (number of ants in each experiment=
%          kA (degree of attraction of branch A)
%          KB (degree of attraction of branch B)
%          n (non-linearity of the selection function)
%Salida: z (number of ants after i+1 ants are on the bridge)
ganadora=zeros(1,i);
zi=zeros;
for L=1:m
z=0;
A=z;
B=z;
vA=zeros(1,i);
vB=zeros(1,i);
for J=0:i-1
z=Deneubourg(z,KA,kB,n,J);
A=z;
B=(J+1)-A;
vA(J+1)=A;
vB(J+1)=B;
end
zi(L)=A;
maxAB=max(A,B);
if(max(A,B)==maxAB)
ganadoraAB=ganadoraAB+vA;
else
ganadoraAB=ganadoraAB+vB;
end
end
fzi=hist(zi,i+1);
ganadoraAB=ganadoraAB./(m*(1:i))*100;
```

III. RESULTS

In Figures 3 to 6 show the results of Monte Carlo analysis for different values of the parameters i (number of ants at each experiment) k_A (degree of attraction of the branch A), K_B (degree of attraction of the branch B) and n (non-linearity of the function selection). All simulations were performed with $m = 10000$ experiments. The figure to the left shows the histogram of the distribution $f(z_{10000})$

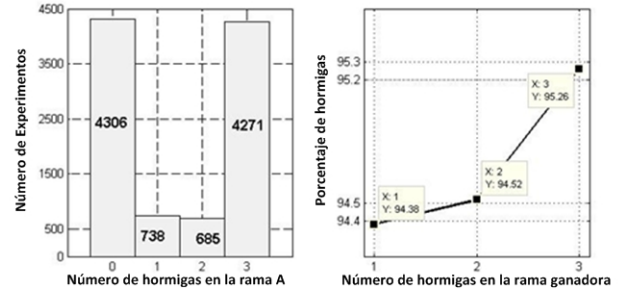


Figure 3. Experiment ant $i = 3$, the values of $K_A = K_B = 1$ was used and $n = 3$

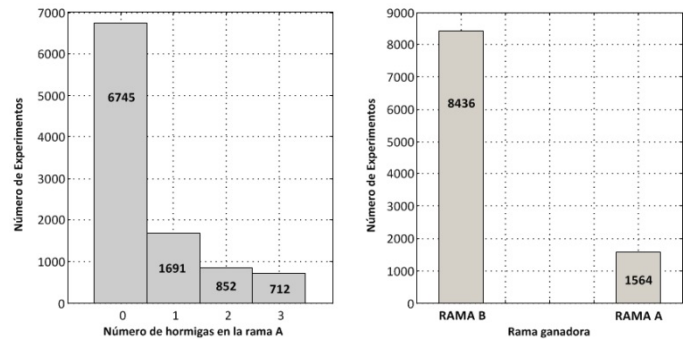


Figure 4. Experiment with $i = 3$ ants, $k_A = 1$ values were used, $K_B = 2$ and $n = 2$.

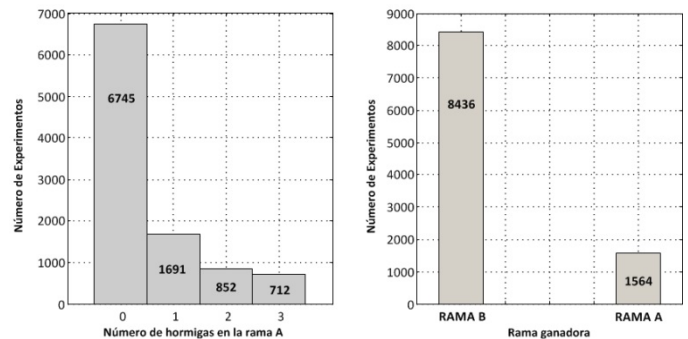


Figure 5. Experiment with $i = 1000$ ants, $K_A = K_B$ values = 10 and $n = 3$ were used. So obtained a sample mean of 499.03 and a sample standard deviation of 484.65

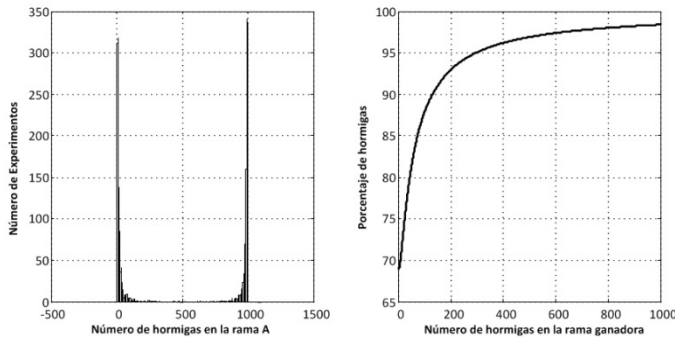


Figure 6. Experiment with $i = 1000$ ants, $K_A = K_B$ values = 100 and $n = 2$. A sample mean of 501.66 was obtained and a standard deviation of 130.53

In Figure 3, note that the percentage of ants on the winning branch, consistent with the results shown in Table 1, where the analysis performed by using equations (2) - (10). Also, note that the frequency histogram for $f(z_3)$ in Figures 3 and 4 are consistent with the data shown in Figure 2. Figure 5 shows how a small attraction parameter $k = 10$, compared with the number of ants bridge used $i = 1000$, high linearity and no selection function $n = 3$ ensure that the percentage of ants on the winning branch is close to 100% when at least a quarter of ants (250 in this case) has used the bridge. In contrast, a parameter of attraction 10 times greater than the previous and nonlinearity causes lower $n = 2$, causes that there were no difference between the winner and the loser branch, shown in the right graph of figure 6 that when a quarter of ants (250 in this case) has used the bridge the percentage of ants in the winning branch is close to 50%.

The resulting percentage of ants on the winning branch can be explained from the histogram of frequencies $f(z)$ (left graph in Figures 5 and 6). Both histograms correspond to symmetric data samples (note that the value of the average is about 500 in both cases), but the histogram in Figure 5 has a higher dispersion compared to the histogram in Figure 6 (note the value of the typical deviation).

IV. CONCLUSIONS

In this paper, the necessary equations were defined to analyze the probability model of Deneubourg for the particular case of $i = 3$ ants. Also, it showed the as Monte Carlo analysis performed using Matlab. The results in both cases were very similar. Furthermore, the importance of the attraction parameter K_A and K_B was observed. If $K_A = K_B = K$ the

probability distribution is symmetrical. If $K \ll i$, then the symmetrical distribution will have a large dispersion, increasing the percentage of ants on the winning branch. On the other hand, if $K = i$, the distribution will have a small dispersion, and the ants will have precedence over any of the branches. Finally, if $K_A < K_B$ distribution becomes anti-symmetric and anti-most often ants would prefer to go by the branch B.

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