An Approach for Switching of the Process Noise Co-Variance of Kalman Filter for Maneuvering Targets

S.M.Sahoo[1], Sandipan Sarkar[2], H.K.Ratha[3]
[1, 2, 3] Integrated Test Range
DRDO
Chandipur – 756 025, India

M.M.Kuber[4]
[4]R & DE (Engrs)
DRDO
PUNE- 411 015, India

Abstract-: Air defence application like tactical weapons system requires that maneuverable vehicles such as aircrafts, ships, submarines and missiles, be tracked accurately. A Linear Kalman Filter has been implemented with switching of process noise co-variance (Q) during the different phases of flight. In this paper switching of Q has been selected by heuristic method or a-prior knowledge of the vehicles dynamics instead of a constant Q. Switching Q technique has come out as a better solution for position and velocity estimation of maneuverable vehicles. The performance of the filter has been evaluated on the basis of Noise Reduction Ratio (NRR).

Keywords-Kalman Filter (KF); Range; Azimuth; Elevation; Noise Reduction Ratio (NRR)

I. INTRODUCTION

Estimation of position, velocity and acceleration using the measurements from radar is essential for the purpose of tracking targets and evaluation of performance of flight vehicles. KF has been popularly used for this purpose. It is the optimal linear estimator for linear Gaussian systems, giving the minimum mean squared error [1]. Using state estimates, the filter can also estimate what the corresponding (output) data are. All realistic measurement systems are affected by noise in their measurements, so accurate tracking can only be achieved by minimizing the impact of random noise connected with the measurements. To accurately estimate target's position or velocity, the estimator needs to know the statistical characteristics of the measurement noise. However KF has the inherent advantage of its easy adaptability to the situation of varying noise characteristics and / or sampling interval. Radar measurements are very much noisy and only give the positional information of the target. But smooth and accurate position and velocity information are required for performance evaluation. To extract smooth position and velocity information, KF has been used with position, velocity and acceleration in Cartesian frame as a state of process model. The measurements for this KF are position in Cartesian frame which are obtained after conversions of radar measurements i.e. range, azimuth and elevation angles. The filter is tuned to get consistent estimate of target state. The KF tuning is to improve the performance of devices based on stochastic filtering theory [2]. The best tuning is achieved using the a-priory knowledge of process noise covariance matrix (Q) and measurement noise covariance matrix (R_m) of filter. Tuning the process noise covariance matrix (Q) and measurement noise covariance matrix (R_m) of filter is very critical in real time scenario. The real time implementation of KF is proposed in the current paper with noise co-variance matrix Q based on [3]. M. Farooq and S Bruder [4] proposed one approach to calculate R_m from spherical frame to Cartesian frame. In this paper we use that formula to get the R_m. This present paper compares the performance of the estimator with static Q and switching Q approach using NRR.

The radar measurement and the KF outputs are in East-North-Vertical (ENV) frame defined by latitude, longitude and height of radar site. Section II gives the details of measurement model. Section III provides the details of the estimator algorithm. Simulation results & analysis of flight trajectory is presented in Section-IV. Finally Section V presents the concluding remarks.

II. RADAR MEASUREMENT

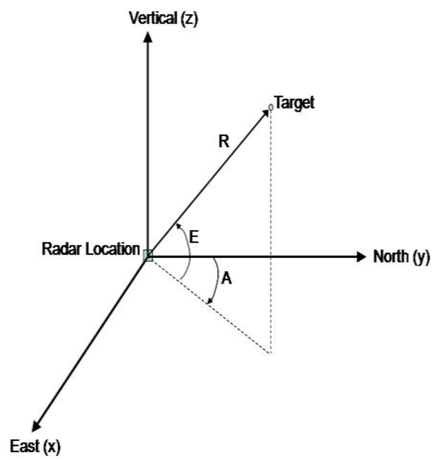

Figure 1: Angle measurement convention of radar in RP frame

Radar measurements are in an ENV frame defined on its location. This ENV frame is refereed as Radar Point (RP) frame. The target measurement in spherical form has been expressed in Range (R), Azimuth (A) and elevation (E). The angle measurements convention used for radar is shown in Fig. 1. Thus the measured target location in Cartesian form is expressed as

$$\begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix}_{RP} = \begin{bmatrix} R\cos E \sin A \\ R\cos E \cos A \\ R \sin E \end{bmatrix} \quad (1)$$

III. METHODOLOGY

A. Kalman Filter Equations

The general form of the kinematic state model of a tracked target is given by

$$x_{(t+1)} = \phi\, x_{(t)} + w_{(t)} \quad (2)$$

The measurements equation is

$$z_{(t+1)} = H\, x_{(t+1)} + v_{(t)} \quad (3)$$

Where $x_{(t)}$ is the target state vector at time t, ϕ is the state transition matrix, z is the measurement vector, H is the observation matrix, $w_{(t)}$ is the process noise uncertainty, $v_{(t)}$ is the measurement noise uncertainty. It is assumed that the noise sequences $w_{(t)}$ and $v_{(t)}$ are zero mean white Gaussian and are mutually independent and have diagonal co-variances $Q(=\sigma_w^2)$ & $R_m(=\sigma_v^2)$ respectively.

B. Propagation Equation

The KF State prediction & State prediction covariance are:

$$\hat{x}_{(t+1/t)} = \Phi\, x_{(t/t)} \quad (4)$$

$$\hat{P}_{(t+1/t)} = \Phi\, P_{(t/t)} \Phi' + Q \quad (5)$$

Where $\hat{x}_{(t+1/t)}$ is the predicted state and $\hat{P}_{(t+1/t)}$ is the state predicted co-variance matrix. In this paper state vector X consist with position, velocity & acceleration with ENV direction.

$$X = [x\ \dot{x}\ \ddot{x}\ y\ \dot{y}\ \ddot{y}\ z\ \dot{z}\ \ddot{z}]^T$$

Where x, y, z are the estimated position components, $\dot{x}, \dot{y}, \dot{z}$ are the estimated velocity components, $\ddot{x}, \ddot{y}, \ddot{z}$ are the estimated acceleration components.

State transition matrix Φ is 9x9 dimensions

$$\phi = \begin{bmatrix} 1 & T & \frac{T^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & \frac{T^2}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & \frac{T^2}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Where T=0.1sec. Initial state co-variance matrix

$$P_0 = \begin{bmatrix} 10^8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10^8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10^8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^8 \end{bmatrix}$$

Process noise covariance matrix with 9x9 dimensions is given below.

Q- [■(■(T↑5/20&T↑4/8&T↑3/6@T↑4/8&T↑3/3&T↑2/2@T↑3/6&T↑2/2&T

Where δ is a multiplying factor.

C. Update Equation

The Kalman gain, updated state & updated covariance of state are:

$$K=\hat{P}_{(t+1/t)}H'(H\hat{P}_{(t+1/t)}\mathbf{H}' + R_m)^{-1} \quad (6)$$

$$x_{(t+1/t+1)}=\hat{x}_{(t+1/t)}+K(Z_{(t+1)} - H\hat{x}_{(t+1/t)}) \quad (7)$$

$$P_{(t+1/t+1)}=[\mathbf{I}-(\mathbf{KH})]\hat{P}_{(t+1/t)}[\mathbf{I}-(\mathbf{KH})]' + \mathbf{K}R_m\mathbf{K}' \quad (8)$$

Where K is the Kalman gain, $X_{(t+1/t+1)}$ is the updated state, $P_{(t+1/t+1)}$ is the updated covariance of state & I is the Identity matrix. Here measurement matrix Z is defined in equation (1). The observation matrix H with 3x9 dimensions is defined as follows:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Measurement co-variance matrix R_m with 3x3 dimensions is derived as follows [4][5]:

$$R_m = dig\{R_c\} \quad (9)$$

$$R_c = JR_pJ' \quad (10)$$

Where R_c is the measurement co-variance in Cartesian frame, J is the Jacobian of the equation (1) and R_p is the measurement co-variance in spherical frame.

$$R_p = \begin{bmatrix} \sigma_R^2 & 0 & 0 \\ 0 & \sigma_A^2 & 0 \\ 0 & 0 & \sigma_E^2 \end{bmatrix}$$

Where σ_R^2 , σ_A^2 and σ_E^2 are the noise characteristics of a radar in Range (R), Azimuth (A) and Elevation (E).

$$J = \begin{bmatrix} \sin A\cos E & R\cos A\cos E & -R\sin A\sin E \\ \cos A\cos E & -R\sin A\cos E & -R\cos A\sin E \\ \sin E & 0 & R\cos E \end{bmatrix}$$

IV. SIMULATION RESULTS

A. Simulation Condition

Here the trajectory data is measured in terms of Range (R), Azimuth (A), and Elevation (E) in Spherical frame of reference. Sample rate has been considered 100ms (dt = 0.1sec). Measured data has been generated with the statistical characteristics as shown in Table: 1.

TABLE: 1
STATISTICAL INFORMATION OF MEASUREMENT ERROR

	Range-Error (m)	**Azimuth-Error(rad)**	**Elevation-Error (rad)**
Standard Deviation(σ)	16.0	0.0015	0.0015

B. Noise Reduction Ratio (N.R.R)

The Noise Reduction Ratio (NRR) is the most commonly used technique to indicate the noise reduction capability. The noise reduction ratio is defined as the ratio of output variance (σ^2_o) to input variance (σ^2_i) of position.

C. Results of first trajectory

TABLE: 2
SHOWS NOISE REDUCTION RATIO OF ESTIMATOR WITH δ OF Q

δ *of Q*	**Noise Reduction Ratio**		
	X	**Y**	**Z**
100	0.2588	0.2553	0.2287
50	0.2432	0.2413	0.2183
5	0.1973	0.2021	0.2182
2.5	0.1861	0.1935	0.2311
1	0.1741	0.1849	0.2511
0.05	0.1574	0.1679	0.3301

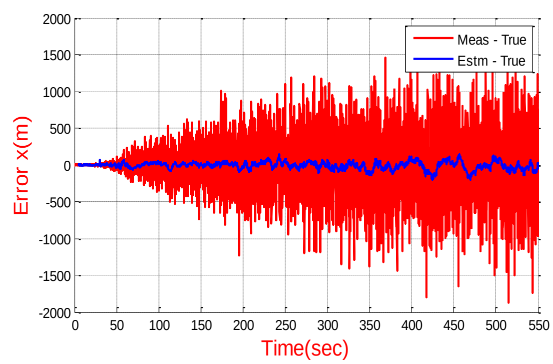

Figure 2.1 Time vs. X error

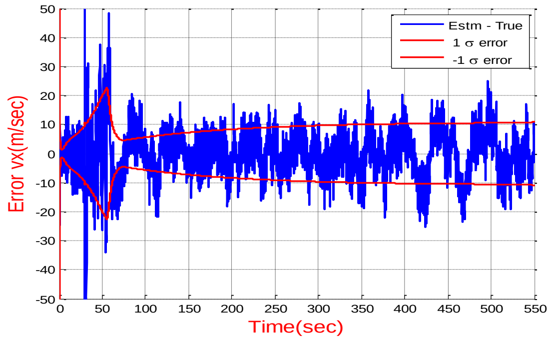

Figure 2.2 Time vs. Velocity error

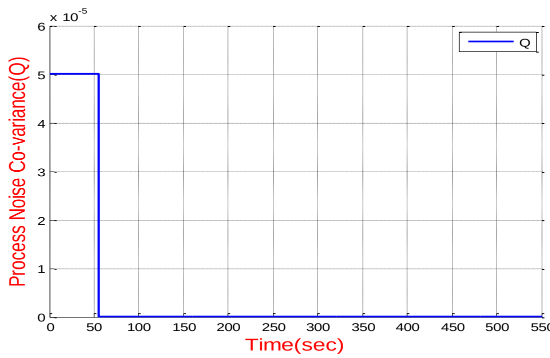

Figure 2.3 Time vs. Q

TABLE: 3
SHOWS NOISE REDUCTION RATIO OF ESTIMATOR WITH δ OF Q WHEN T <= 55 SEC

δ of Q	Noise Reduction Ratio		
	X	Y	Z
100	**0.5228**	**0.4745**	**0.6417**
0.05	1.1165	1.0864	2.4638

TABLE: 4
SHOWS NOISE REDUCTION RATIO OF ESTIMATOR WITH δ OF Q WHEN T > 55 SEC

δ of Q	Noise Reduction Ratio		
	X	Y	Z
100	0.2587	0.2551	0.2285
0.05	**0.1568**	**0.1669**	**0.3290**

TABLE: 5
SHOWS NOISE REDUCTION RATIO OF ESTIMATOR WITH δ OF Q

δ of Q	Noise Reduction Ratio		
	X	Y	Z
δ=100 when t <= 55sec & δ=0.05 when t >55 sec	**0.1569**	**0.1673**	**0.3293**

D. Results of second trajectory

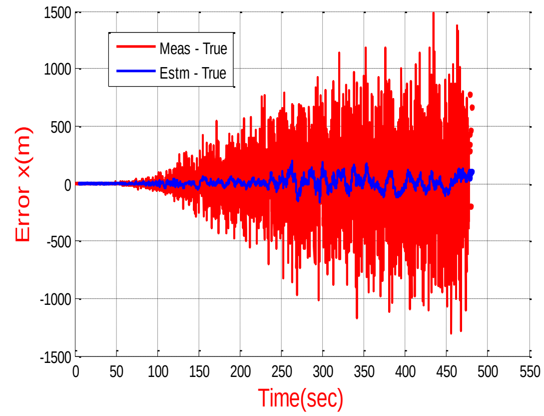

Figure 3.1 Time vs. X error

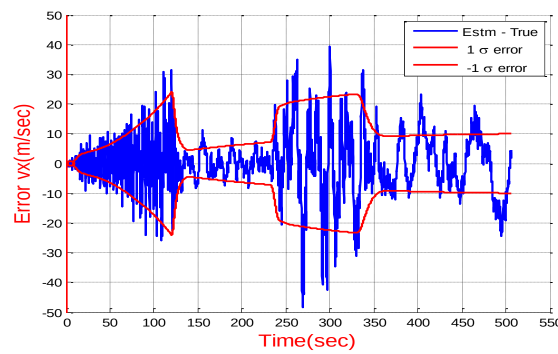

Figure 3.2 Time vs. Velocity error

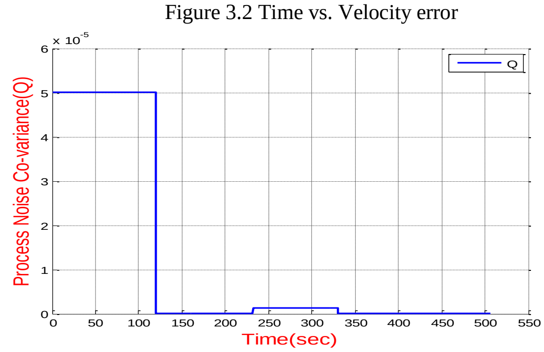

Figure 3.3 Time vs. Q

TABLE: 6
SHOWS NOISE REDUCTION RATIO OF ESTIMATOR WITH δ OF Q

δ of Q	Noise Reduction Ratio		
	X	Y	Z
100	0.2808	0.3067	0.2563
2.5	0.2099	0.2392	0.1960
0.05	0.1618	0.3074	0.4541
Switching of δ 100, 2.5 & 0.05	**0.1606**	**0.1952**	**0.3042**

"Fig. 2.1" shows the plot of time vs. x error and "Fig. 2.2" shows the plot of time vs. velocity error of first trajectory. "Table 2" shows the performance of estimator with different δ value of Q matrix and it concludes that estimator performance is best when the value of δ is 0.05 in X and Y direction and when the value of δ is 5 in Z direction. “Table 3” shows the performance of the estimator from time 0 to 55 sec and “Table 4” shows the performance of estimator from time 55 sec to end of the trajectory. “Table 3” concludes that when velocity has changed sharply then choice of δ value should be very high. “Table 4” concludes that when velocity has changed slowly then choice of δ value should be very less. “Table 5” shows the performance of the estimator with switching the δ value of Q from 100 to 0.05 at 55 sec and we get best result with respect of “Table 2”. Same approach used in trajectory 2 where velocity has changed in different phases of the trajectory. Time vs. X error and time vs. velocity error shown in “Fig. 3.1 & 3.2”. “Table 6” shows the performance of the estimator and it is observed that best performance has been achieved with switching the δ value of Q at different phases of trajectory.

CONCLUSION

This proposed approach has been tested in different maneuvering targets in real time and observed that it is working better in comparison to single Q estimation approach. It may be noted that after sharp change in velocity known as the thrusting phase, the target had undergone very small change in velocity i.e. non- thrusting phase. This necessitates different levels of process noise variance to match the distinct dynamics of the target during thrusting and non-thrusting phase of the flight.

REFERENCES

[1] Morris J. M., 1976 *“The kalman filter: a robust estimator for some classes of linear quadratic problems”*, IEEE Transactions on Information Theory, **22**, pp.526–534.

[2] Yaakov BarShalom, X.Rongli and Thiagalingam Kirubarajan, 2001, “ Estimation with Applications to Tracking and Navigation”, John Wiley & Sons, INC. USA, chap.5.

[3] Eli Brookner, 1998, “Tracking and Kalman Filtering Made Easy”, John Wiley & Sons, INC. USA, chap.2.

[4] M. Farooq and S Bruder, “Information type filters for tracking a maneuvering target”, IEEE Transactions on Aerospace and Electronic Systems, Vol. 39, No. 4, 2003, pp. 1333-1364.

[5] Li-Wei-Fong, “Distributed Data Fusion Algorithms for Tracking a Maneuvering Target”, IEEE Conference on Information Fusion, 2007, pp.1-8.

[6] Maybeck P.S., 1982, “ Stochastic Models, Estimation and Control, Academic Press, NewYork, pp.1-271.