

Resource Optimization for Wireless Networks

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Abstract— In this paper, we investigate the resource optimization problem in MIMO multi-hop wireless networks to maximize throughput with given resource constraints. We propose algorithms with low complexity to achieve maximum capacity. The main idea is to determine the rank of the optimal transmit covariance matrix and the optimal power allocation of each node separately using our low-complexity algorithm. In addition, we reduce the complexity of our algorithm by adding another preprocessing algorithm. Using our algorithms, we find that while dynamical allocation of time and power could increase channel capacity, equal time and power allocation among different nodes may not cause much capacity loss. Numerical results confirm our finding.

Key Words— MIMO, multi-hop wireless networks, capacity analysis, power allocation, resource optimization

I. INTRODUCTION

In cellular and wireless local area networks, wireless communication only occurs on the last link between a base station and the wireless end system. In multihop wireless networks, there are one or more intermediate nodes along the path that receive and forward packets via wireless links. Multihop wireless networks have several benefits: Compared with networks with single wireless links, multihop wireless networks can extend the coverage of a network and improve connectivity. Moreover, transmission over multiple short links might require less transmission power and energy than that required over long links. Moreover, they enable higher data rates resulting in higher throughput and more efficient use of the wireless medium. Multihop wireless networks avoid wide deployment of cables and can be deployed in a cost-efficient way. In case of dense multihop networks, several paths might become available that can be used to increase robustness of the network [1]–[5].

Nowadays multiple-input-multiple-output (MIMO) has been widely used in wireless networks to enhance the capacity, thus new problems appear on MIMO multi-hop channels [12] [13]. Multiple input multiple-output (MIMO) systems are a natural extension of developments in antenna array communication. While the advantages of multiple receive antennas, such as gain and spatial diversity, have been known and exploited for some time, the use of transmit diversity has only been investigated recently. The advantages of MIMO communication, which exploits the physical channel between many transmit and receive antennas, are currently receiving significant attention. While the channel can be so nonstationary that it cannot be estimated in any useful sense, in this article we assume the channel is quasistatic [6]–[11].

MIMO systems provide a number of advantages over single antenna- to-single-antenna communication. Sensitivity to fading is reduced by the spatial diversity provided by multiple spatial paths. Under certain environmental conditions, the power requirements associated with high spectral-efficiency communication can be significantly reduced by avoiding the compressive region of the information-theoretic capacity bound. Here, spectral efficiency is defined as the total number of information bits per second per Hertz transmitted from one array to the other.

We investigate the resource allocation in MIMO multi-hop channels in this paper. We want to maximize the throughput of the whole multi-hop channel so we need to optimize the allocation of the total power and time slots. For the power allocation, we consider two constraints of a total sum power budget and a maximum power per hop [14] [15]. The total sum power constraint corresponds to the maximum sum power that a given packet is allowed to consume throughout its propagation from source to destination, while the maximum power per hop corresponds to the maximum power that each relay node can provide. Given a total sum power constraint, how to allocate power budget with low complexity to each node to realize maximum throughput is discussed.

Before, in order to derive the optimal power allocation of each node, one needed to determine the power allocation and the rank of the optimal transmit covariance matrix jointly. In addition, one may need to solve non-linear equations for exponential times, which leads to a high complexity. We propose an algorithm to reduce the algorithm complexity by determining the rank of the optimal transmit covariance matrix and power allocation separately. Also we reduce the number of non-linear equations from an exponential order to only one non-linear equation. For the time allocation, we discuss both the fixed and adaptive time allocation. The transmission from the transmitter to the receiver via a number of intermediate nodes can operate in a time division multiplex (TDM) manner using a series of time slots. The case using equal time durations is referred to as the fixed time allocation scheme while the case using time slots of different time durations is referred to as the adaptive time allocation scheme [16]. From the perspective of end-to-end throughput maximization, we show the difference between these two schemes.

The rest of this paper is organized as follows. In Section II, we present our system model. In Section III, we discuss the resource allocation for a system with fixed time division. Section

IV discusses similar problems for a system with adaptive time

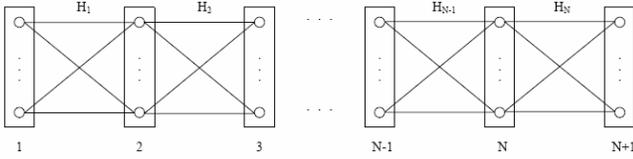


Fig. 1. MIMO N -Hop Fading Channel

division. In Section V, we present some simulation results and summarize the effectiveness of our algorithm. Finally, Section VI contains some concluding remarks.

II. SIGNAL MODEL AND PROBLEM STATEMENT

We consider an N -hop transmission system, where there are one transmit node, one receive node, and $N - 1$ intermediate nodes as shown in Figure 1. Each node is equipped with m antennas. We assume that signals from the transmit node arrive at the receive node via $N - 1$ intermediate nodes. All transmissions are assumed to be half duplex and therefore a node cannot transmit and receive during the same time period. Avoidance of interference suggests a time-division transmission strategy: In every transmission cycle, there are N time slots. At the first time slot, Node 1 transmits signals to intermediate Node 2. Node 2 performs the decode and forward operation. All the following intermediate nodes operate in the same way, until the signals from Node 1 arrive at Node $N + 1$. Here we assume that the transmission cycle is equal to the coherence time T during which all channel matrices are constant. This system works under the assumption of perfectly synchronized transmission and reception among all the nodes and a quasi-static flat Rayleigh fading channel model. The path gains are fixed during the transmission of one block. Perfect channel information is known by all the nodes. \mathbf{H}_i and C_i denote the channel matrix and channel capacity between Node i and Node $i + 1$. Let \mathbf{x}_i and \mathbf{y}_{i+1} denote the transmitted signals of node i and the received signals of node $i + 1$, $i = 1, \dots, N$, respectively. So we have

$$\mathbf{y}_{i+1} = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_i \quad (1)$$

where \mathbf{n}_i is a zero-mean, identity-covariance complex Gaussian vector, and the entries of \mathbf{H}_i are complex Gaussian random variables. Let $\mathbf{Q}_i = E[\mathbf{x}_i \mathbf{x}_i^\dagger]$ be the transmit covariance matrix of Node i . For any given channel realization \mathbf{H}_i , the instantaneous channel capacity between Node i and Node $i + 1$ can be derived as [17]

$$C_i = \log \det(\mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^\dagger + \mathbf{I}) \text{ bps/Hz}. \quad (2)$$

Suppose every node works in the same frequency band with bandwidth W . The total frame duration T is partitioned into slots T_i , $i = 1, \dots, N$, attributed to the transmit node and all the intermediate nodes, i.e., $\sum_{i=1}^N T_i = T$. So the total bits transmitted from Node i to Node $i + 1$ during time slot T_i is given by $\text{Throughput}_i = W \times T_i \times C_i$, where C_i is calculated by (2). Since the channel matrix \mathbf{H}_i and transmit covariance matrix \mathbf{Q}_i for different neighboring nodes are not the same, the channel capacity C_i are also different.

Note that the end-to-end throughput is determined by the smallest Throughput_i [17]. In order to maximize the end-to-end throughput in coherence time T , we need to optimize the time partition T_i and channel capacity C_i . This is our main objective. Note that C_i is determined by \mathbf{Q}_i once \mathbf{H}_i is fixed. We assume P_i , $i = 1, \dots, N$ are the power allocated to the transmit node and all the intermediate nodes, i.e., $\text{tr}(\mathbf{Q}_i) = P_i$. We assume the total sum power constraint of the transmit node and all the intermediate nodes is P , i.e., $\sum_{i=1}^N P_i \leq P$ [15].

III. RESOURCE ALLOCATION WITH FIXED TIME DIVISION

According to our time division strategy, the transmit node and all the intermediate nodes transmit signals in consecutive time slots. At any time slot, only one node transmits information. So there is no co-channel interference during the entire transmission. In addition, the end-to-end throughput equals to the smallest Throughput_i , $i = 1, \dots, N$. So the end-to-end throughput is maximized when every Throughput_i is equal and maximized, i.e.,

$$W \cdot T_1 \cdot C_1 = W \cdot T_2 \cdot C_2 = \dots = W \cdot T_N \cdot C_N = C \text{ bits}. \quad (3)$$

In this section, we adopt the fixed time division strategy, i.e., the time slots allocated to the transmit node and all the intermediate nodes are equal. Then we have

$$C_1 = C_2 = \dots = C_N. \quad (4)$$

Therefore, we can simply maximize C_i , $i = 1, \dots, N$, to obtain the largest total throughput. Let the eigen-decomposition of $\mathbf{H}_i^\dagger \mathbf{H}_i$ be

$$\mathbf{H}_i^\dagger \mathbf{H}_i = \mathbf{U}_i^\dagger \mathbf{\Lambda}_i \mathbf{U}_i = \mathbf{U}_i^\dagger \text{diag}\{\lambda_{i1}, \lambda_{i2}, \dots, \lambda_{im}\} \mathbf{U}_i. \quad (5)$$

It has been known that \mathbf{Q}_i maximizing C_i should have the following structure [18]

$$\mathbf{Q}_i = \mathbf{U}_i^\dagger \mathbf{\Omega}_i \mathbf{U}_i = \mathbf{U}_i^\dagger \text{diag}\{\gamma_{i1}, \gamma_{i2}, \dots, \gamma_{im}\} \mathbf{U}_i \quad (6)$$

where

$$\gamma_{ij} = (\mu_i - \lambda_{ij}^{-1})^+, j = 1, \dots, m \quad (7)$$

and μ_i is chosen to satisfy

$$\sum_{j=1}^m \gamma_{ij} = P_i. \quad (8)$$

Then (2) could be written as

$$C_i = \log \det(\mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^\dagger + \mathbf{I}) = \log \prod_{j=1}^m (\lambda_{ij} \gamma_{ij} + 1) \quad (9)$$

Combining (4), (7), (9), we derive the following $N + 1$ equations

$$\prod_{j=1}^m (\lambda_{ij} \gamma_{ij} + 1) = c, \quad i = 1, \dots, N \quad (10)$$

$$\sum_{i=1}^N \sum_{j=1}^m \gamma_{ij} = P \quad (11)$$

where c is a constant.

Lemma 1: Let us denote the eigenvalues of $\mathbf{H}_i^\dagger \mathbf{H}_i$ by $\lambda_{i1} < \lambda_{i2} < \dots < \lambda_{im}$, then the optimal \mathbf{Q}_i has n ($n < m$) non-zero eigenvalues if and only if $\{\lambda_{ij}\}$ satisfy

$$n\lambda_{i,m-n+1}^{-1} < \sum_{j=1}^n \lambda_{i,m-j+1}^{-1} + P_i < n\lambda_{i,m-n}^{-1} \quad (12)$$

or m non-zero eigenvalues if and only if $\{\lambda_{ij}\}$ satisfy

$$\sum_{j=1}^m \lambda_{i,m-j+1}^{-1} + P_i > m\lambda_{i1}^{-1} \quad (13)$$

Proof: After substituting (7) in (8), we can derive the above results easily. \square

Using Lemma 1, we prove the following theorem.

Theorem 1: For $i = 1, \dots, N$, if each \mathbf{Q}_i has n non-zero eigenvalues γ_{ij} , $j = 1, \dots, n$, which correspond to λ_{ij} , $j = 1, \dots, n$, respectively, then the optimal power allocation for all the nodes is given by

$$P_i = n\mu_i - \sum_{j=1}^n \lambda_{i,m-j+1}^{-1}, \quad i = 1, \dots, N \quad (14)$$

where

$$\mu_i = \frac{P + \sum_{k=1}^N \sum_{j=1}^n \lambda_{k,m-j+1}^{-1}}{\sum_{k=1}^N \sqrt{\frac{\prod_{j=1}^n \lambda_{i,m-j+1}}{\prod_{j=1}^n \lambda_{k,m-j+1}}}} \quad (15)$$

Proof: Substitute (7) into (10) (11) and solve the equations. We can derive each μ_i , γ_{ij} and further P_i . \square

As a special case of Theorem 1, we have:

Corollary 1: If $\lambda_{i,m-1}^{-1} - \lambda_{i,m}^{-1} > P_i$, then the best transmission scheme to maximize the total throughput is that every node adopts one-dimensional beamforming, and the optimal power allocation is given by

$$P_i = \frac{P + \sum_{k=1}^N \lambda_{km}^{-1}}{\sum_{k=1}^N \frac{\lambda_{im}}{\lambda_{km}}} - \lambda_{im}^{-1}, \quad i = 1, \dots, N \quad (16)$$

However, when \mathbf{Q}_i , $i = 1, \dots, N$, do not have the same number of non-zero eigenvalues, we must calculate them by the following theorem.

Theorem 2: For $i = 1, \dots, N$, if \mathbf{Q}_i has n_i non-zero eigenvalues γ_{ij} , $j = 1, \dots, n_i$, which correspond to λ_{ij} , $j = 1, \dots, n_i$, respectively. Then, the optimal power allocation for all the nodes is given by

$$P_i = n_i \mu_i - \sum_{j=1}^{n_i} \lambda_{i,m-j+1}^{-1} \quad (17)$$

where μ_i is given by the solution of the following non-linear equation

$$\sum_{k=1}^N n_k \left(\frac{\prod_{j=1}^{n_i} \lambda_{i,m-j+1}}{\prod_{j=1}^{n_k} \lambda_{k,m-j+1}} \right)^{\frac{1}{n_k}} \mu_i^{\frac{n_i}{n_k}} = P + \sum_{k=1}^N \sum_{j=1}^{n_k} \lambda_{k,m-j+1}^{-1}. \quad (18)$$

Proof: Similar to the proof of Theorem 1. \square

From Theorem 1 and Theorem 2, we can see that in order to calculate P_i , we must know how many non-zero eigenvalues each \mathbf{Q}_i has, i.e., the rank of \mathbf{Q}_i . But if we do not know P_i , we cannot determine the rank of \mathbf{Q}_i . So we need to find P_i and

the rank of \mathbf{Q}_i jointly. A straightforward way is the following algorithm.

Algorithm 1:

- Step 1: Give initial values to n_i , $i = 1, \dots, N$.
- Step 2: Calculate P_i using Theorem 1 or 2.
- Step 3: Check whether P_i derived satisfies Lemma 1.
- Step 4: If P_i derived satisfies Lemma 1, then stop. Otherwise, repeat Step 1 to Step 3 with new values for n_i until the final P_i is found.

Obviously, by this algorithm, we need to solve the non-linear equation for m^N times at most, which is time-consuming. So we aim to reduce the calculation complexity in the next subsection.

A. Reducing the Calculation Complexity of Algorithm 1

First, note that the calculation complexity mainly comes from the m^N times trial at most in order to find optimal n_i , the rank of the optimal \mathbf{Q}_i , for each node. In addition, in each trial, we need to solve a non-linear equation. So we have to solve non-linear equations m^N times in the worst case. We propose the following algorithm, in which instead of solving n_i and P_i jointly, we first derive the n_i through at most mN times trial, then we solve only one non-linear equation to get P_i .

Algorithm 2:

- Step 1: Initialize $i = 1$. Let $\alpha_i = 1$ and $\beta_i = m$.
- Step 2: Let $\mu_i = \lambda_{i,m-\alpha_i+1}^{-1}$, use (7)(10) to calculate $\mu_{i'}$, $i' = 1, \dots, N$, $i' \neq i$. Then substitute all $\mu_{i'}$, $i' = 1, \dots, N$, to $\sum_{i'=1}^N \sum_{j=1}^m \gamma_{i'j}$. If $\sum_{i'=1}^N \sum_{j=1}^m \gamma_{i'j} < P$ and $\alpha_i < \beta_i$, let $\alpha_i = \alpha_i + 1$ and repeat this step. If $\sum_{i'=1}^N \sum_{j=1}^m \gamma_{i'j} < P$ and $\alpha_i = \beta_i$, let $n_i = \beta_i$ and go to Step 3. If $\sum_{i'=1}^N \sum_{j=1}^m \gamma_{i'j} > P$, let $n_i = \alpha_i - 1$ and go to Step 3.
- Step 3: Let $i = i + 1$, $\alpha_i = 1$, $\beta_i = m$. Repeat Step 2 until all n_i , $i = 1, \dots, N$, are derived.
- Step 4: Calculate P_i using Theorem 2.

Here, we have

Lemma 2: The optimal \mathbf{Q}_i , $i = 1, \dots, N$, has n_i non-zero eigenvalues exactly.

Proof: Let $\hat{\alpha}_i$, denote the number of non-zero eigenvalues of the optimal \mathbf{Q}_i , $i = 1, \dots, N$. $\hat{\mu}_i$, $i = 1, \dots, N$, denote the water-filling level corresponding to the optimal \mathbf{Q}_i , $i = 1, \dots, N$. Then $\hat{\mu}_i$, $i = 1, \dots, N$ should satisfy

$$\hat{c} = \hat{\mu}_i^{\hat{\alpha}_i} \prod_{j=1}^{\hat{\alpha}_i} \lambda_{i,m-j+1}, \quad i = 1, \dots, N. \quad (19)$$

$$\sum_{i=1}^N \hat{s}_i = P, \quad \hat{s}_i = \hat{\alpha}_i \hat{\mu}_i - \sum_{j=1}^{\hat{\alpha}_i} \lambda_{i,m-j+1}^{-1} \quad (20)$$

Without loss of generality, we take \mathbf{Q}_1 for example.

In Step 2 of Algorithm 2, if we let $\mu_1 = \lambda_{1,m-\hat{\alpha}_1+k}^{-1}$, where k is integer and $k > 0$, since $\mu_1 < \hat{\mu}_1$, we have $c < \hat{c}$ and $s_1 < \hat{s}_1$. For all other $i = 2, \dots, N$, since $c < \hat{c}$, we have $s_i < \hat{s}_i$. So we have $\sum_{i=1}^N \sum_{j=1}^m \gamma_{ij} < P$.

In Step 2 of Algorithm 2, if we let $\mu_1 = \lambda_{1,m-\hat{\alpha}_1}^{-1}$, since $\mu_1 > \hat{\mu}_1$, we have $c > \hat{c}$ and $s_1 > \hat{s}_1$. For $i = 2, \dots, N$, since $c > \hat{c}$, we have $s_i > \hat{s}_i$. So we have $\sum_{i=1}^N \sum_{j=1}^m \gamma_{ij} > P$.

Therefore, n_1 derived in Algorithm 2 is the rank of the optimal \mathbf{Q}_i exactly. Similarly, $n_i, i = 2, \dots, N$, derived in Algorithm 2 is the rank of the optimal \mathbf{Q}_i exactly. \square

Note that in Algorithm 2, in order to get n_i , the rank of the optimal \mathbf{Q}_i , for each node, we assume μ_i is equal to each $\lambda_{i,j}, j = 1, \dots, m$ and calculate the resulting total power $\sum_{i'=1}^N \sum_{j=1}^m \gamma_{i'j}$. By comparing $\sum_{i'=1}^N \sum_{j=1}^m \gamma_{i'j}$ with the actual total power P , we can derive n_i . Then we follow the same steps to get n_{i+1} and so on. In this way, we reduce the number of non-linear equations we need to solve from m^N to only 1. In addition, instead of m^N times trial to get n_i and P_i simultaneously, now we only need to try mN times to get the n_i . And in each trial, only simple linear calculations are needed. So the complexity to derive P_i is reduced significantly.

In order to further reduce the complexity of calculating P_i , we propose the following algorithm that can be added before Algorithm 2.

Algorithm 3:

- Step 1: Assume all n_i are equal to 1 and calculate $\{\mu_i^{(1)}\}, i = 1, \dots, N$, using Theorem 1. In the same way, assume all n_i are equal to $n, n = 2, \dots, N$, and calculate $\{\mu_i^{(n)}\}, i = 1, \dots, N$.
- Step 2: For each node i , find the largest n for which $\mu_i^{(n)} > \lambda_{i,m-n+1}^{-1}$. Let $\alpha = n$. Find the smallest n for which $\mu_i^{(n)} < \lambda_{i,m-n+1}^{-1}$. Let $\beta = n - 1$. If there is no n for which $\mu_i^{(n)} < \lambda_{i,m-n+1}^{-1}$, let $\beta = m$.
- Step 3: For each node i , if $\mu_i^{(\alpha)} > \lambda_{i,m-\alpha}^{-1}$, let $\alpha_i = \alpha + 1$. Otherwise, let $\alpha_i = \alpha$. If $\mu_i^{(\beta)} < \lambda_{i,m-\beta+1}^{-1}$, let $\beta_i = \beta - 1$. Otherwise, let $\beta_i = \beta$.

Now, we have the following two lemmas.

Lemma 3: The optimal $\mathbf{Q}_i, i = 1, \dots, N$, has no less than α_i non-zero eigenvalues.

Proof: Without loss of generality, we take \mathbf{Q}_{i_0} for example, where $i_0 \in \Phi = \{1, \dots, N\}$. Let $\hat{\alpha}_{i_0}$ denote the number of non-zero eigenvalues of the optimal \mathbf{Q}_{i_0} . We first prove $\hat{\alpha}_{i_0} \geq \alpha$, then we prove $\hat{\alpha}_{i_0} \geq \alpha_i$. Now we assume $\hat{\alpha}_{i_0} < \alpha$. Next, we will prove that this assumption cannot stand.

By Algorithm 3, $\alpha = 1$ or $\alpha > 1$. If $\alpha = 1$, it is obvious that $\hat{\alpha}_{i_0} < \alpha$ cannot stand. If $\alpha > 1, \mu_i^{(\alpha)}, i = 1, \dots, N$, derived in Algorithm 3 satisfy

$$c = \mu_i^\alpha \prod_{j=1}^{\alpha} \lambda_{i,m-j+1}, \quad i = 1, \dots, N. \quad (21)$$

$$\sum_{i=1}^N s_i = P, \quad s_i = \alpha \mu_i - \sum_{j=1}^{\alpha} \lambda_{i,m-j+1}^{-1} \quad (22)$$

where we neglect the superscript (α) of $\mu_i^{(\alpha)}$ for simplification.

Let $\hat{\alpha}_i, i = 1, \dots, N$, denote the number of non-zero eigenvalues of the optimal $\mathbf{Q}_i, i = 1, \dots, N$. $\hat{\mu}_i, i = 1, \dots, N$, denote the water-filling level corresponding to the optimal $\mathbf{Q}_i, i = 1, \dots, N$. Then $\hat{\mu}_i, i = 1, \dots, N$ should satisfy (19) and (20).

For $i = i_0$, since we have assumed $\hat{\alpha}_{i_0} < \alpha$, we have $\hat{\mu}_{i_0} < \lambda_{i_0,m-\hat{\alpha}_{i_0}}^{-1} \leq \lambda_{i_0,m-\alpha+1}^{-1} < \mu_{i_0}$. By comparing (21) with (19), we have $c > \hat{c}$. By comparing (22) with (20), we have $s_{i_0} > \hat{s}_{i_0}$.

We partition Φ not including element i_0 into three subsets, $\Phi_1 = \{i_1\}, \Phi_2 = \{i_2\}, \Phi_3 = \{i_3\}$. For any element $i_1 \in \Phi_1$, the optimal \mathbf{Q}_{i_1} has $\hat{\alpha}_{i_1}$ non-zero eigenvalues and $\hat{\alpha}_{i_1} = \alpha$. For any element $i_2 \in \Phi_2$, the optimal \mathbf{Q}_{i_2} has $\hat{\alpha}_{i_2}$ non-zero eigenvalues and $\hat{\alpha}_{i_2} < \alpha$. For any element $i_3 \in \Phi_3$, the optimal \mathbf{Q}_{i_3} has $\hat{\alpha}_{i_3}$ non-zero eigenvalues and $\hat{\alpha}_{i_3} > \alpha$.

For $i = i_1 \in \Phi_1$, since we have known that $c > \hat{c}$ and $\hat{\alpha}_{i_1} = \alpha$, by comparing (21) with (19), we have $\mu_{i_1} > \hat{\mu}_{i_1}$. By comparing (22) with (20), we have $s_{i_1} > \hat{s}_{i_1}$.

For $i = i_2 \in \Phi_2$, since $\hat{\alpha}_{i_2} < \alpha$, we have $\hat{\mu}_{i_2} < \lambda_{i_2,m-\hat{\alpha}_{i_2}}^{-1} \leq \lambda_{i_2,m-\alpha+1}^{-1} < \mu_{i_2}$. By comparing (22) with (20), we have $s_{i_2} > \hat{s}_{i_2}$.

From (22) and (20), we know that

$$s_{i_0} + \sum_{i_1 \in \Phi_1} s_{i_1} + \sum_{i_2 \in \Phi_2} s_{i_2} + \sum_{i_3 \in \Phi_3} s_{i_3} = P \quad (23)$$

$$\hat{s}_{i_0} + \sum_{i_1 \in \Phi_1} \hat{s}_{i_1} + \sum_{i_2 \in \Phi_2} \hat{s}_{i_2} + \sum_{i_3 \in \Phi_3} \hat{s}_{i_3} = P \quad (24)$$

Since we have known that $s_{i_0} > \hat{s}_{i_0}, s_{i_1} > \hat{s}_{i_1}$ and $s_{i_2} > \hat{s}_{i_2}$, from (23) and (24), we know that $s_{i_3} < \hat{s}_{i_3}$. Next, we will prove that $s_{i_3} < \hat{s}_{i_3}$ cannot stand.

For $i = i_3 \in \Phi_3$, since we know that $c > \hat{c}$, by (21) and (19), we have

$$\mu_{i_3}^\alpha \prod_{j=1}^{\alpha} \lambda_{i_3,m-j+1} > \hat{\mu}_{i_3}^{\hat{\alpha}_{i_3}} \prod_{j=1}^{\hat{\alpha}_{i_3}} \lambda_{i_3,m-j+1} \quad (25)$$

$$\text{i.e., } \mu_{i_3} > \hat{\mu}_{i_3}^{\frac{\hat{\alpha}_{i_3}}{\alpha}} \left(\prod_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3,m-j+1} \right)^{\frac{1}{\alpha}}$$

Suppose $s_{i_3} < \hat{s}_{i_3}$, by (22) and (20), we have

$$\alpha \mu_{i_3} - \sum_{j=1}^{\alpha} \lambda_{i_3,m-j+1}^{-1} < \hat{\alpha}_{i_3} \hat{\mu}_{i_3} - \sum_{j=1}^{\hat{\alpha}_{i_3}} \lambda_{i_3,m-j+1}^{-1} \quad (26)$$

$$\text{i.e., } \mu_{i_3} < \frac{\hat{\alpha}_{i_3}}{\alpha} \hat{\mu}_{i_3} - \frac{1}{\alpha} \sum_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3,m-j+1}^{-1}$$

Now we will prove that (25) and (26) cannot stand simultaneously. Let

$$f(\hat{\mu}_{i_3}) = \hat{\mu}_{i_3}^{\frac{\hat{\alpha}_{i_3}}{\alpha}} \left(\prod_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3,m-j+1} \right)^{\frac{1}{\alpha}} - \frac{\hat{\alpha}_{i_3}}{\alpha} \hat{\mu}_{i_3} + \frac{1}{\alpha} \sum_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3,m-j+1}^{-1} \quad (27)$$

Since $\frac{\partial^2 f(\hat{\mu}_{i_3})}{\partial \hat{\mu}_{i_3}^2} > 0$, $f(\hat{\mu}_{i_3})$ has a minimum value.

Let $\frac{\partial f(\hat{\mu}_{i_3})}{\partial \hat{\mu}_{i_3}} = 0$, we can calculate that when $\hat{\mu}_{i_3} = \left(\prod_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3,m-j+1} \right)^{\frac{1}{\alpha-\hat{\alpha}_{i_3}}}$, $f(\hat{\mu}_{i_3})$ has the minimum value

$$\begin{aligned}
 f(\hat{\mu}_{i_3}) &= \left(\prod_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3, m-j+1} \right)^{\frac{1}{\alpha-\hat{\alpha}_{i_3}}} \\
 &- \frac{\hat{\alpha}_{i_3}}{\alpha} \left(\prod_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3, m-j+1} \right)^{\frac{1}{\alpha-\hat{\alpha}_{i_3}}} + \frac{1}{\alpha} \sum_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3, m-j+1}^{-1} \\
 &= \frac{1}{\alpha} \sum_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3, m-j+1}^{-1} \\
 &- \frac{1}{\alpha} (\hat{\alpha}_{i_3} - \alpha) \left(\prod_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3, m-j+1} \right)^{\frac{1}{\alpha-\hat{\alpha}_{i_3}}} > 0 \quad (28)
 \end{aligned}$$

where we have used the inequality $\sum_{i=1}^n x_i \geq n \sqrt[n]{\prod_{i=1}^n x_i}$. Equation (28) means that

$$\hat{\mu}_{i_3}^{\frac{\hat{\alpha}_{i_3}}{\alpha}} \left(\prod_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3, m-j+1} \right)^{\frac{1}{\alpha}} > \frac{\hat{\alpha}_{i_3}}{\alpha} \hat{\mu}_{i_3} - \frac{1}{\alpha} \sum_{j=\alpha+1}^{\hat{\alpha}_{i_3}} \lambda_{i_3, m-j+1}^{-1} \quad (29)$$

So (25) and (26) cannot be satisfied simultaneously. Therefore, the assumption $\hat{\alpha}_{i_0} < \alpha$ is not correct. So we get the conclusion that $\hat{\alpha}_{i_0} \geq \alpha$.

Now we prove $\hat{\alpha}_{i_0} \geq \alpha_i$. From the above proof, we know that $c < \hat{c}$. If the optimal \mathbf{Q}_{i_0} has α non-zero eigenvalues, then $\mu_{i_0}^{(\alpha)} < \hat{\mu}_{i_0}^{(\alpha)} < \lambda_{i_0, m-\alpha}^{-1}$. So if $\mu_{i_0}^{(\alpha)} > \lambda_{i_0, m-\alpha}^{-1}$, the optimal \mathbf{Q}_{i_0} has more than α non-zero eigenvalues, i.e., the optimal \mathbf{Q}_{i_0} has no less than α_{i_0} non-zero eigenvalues. \square

Lemma 4: The optimal \mathbf{Q}_i , $i = 1, \dots, N$, has no more than β_i non-zero eigenvalues.

Proof: Similar to the proof of Lemma 3, we first consider β then consider β_i . By Algorithm 3, $\beta = m$ or $\beta < m$. If $\beta = m$, it is obvious that the optimal \mathbf{Q}_i has no more than β non-zero eigenvalues. If $\beta < m$, $\mu_i^{(\beta+1)}$, $i = 1, \dots, N$, derived in Algorithm 3 satisfy

$$c = \mu_i^{\beta+1} \prod_{j=1}^{\beta+1} \lambda_{i, m-j+1}, \quad i = 1, \dots, N. \quad (30)$$

$$\sum_{i=1}^N s_i = P, \quad s_i = (\beta+1)\mu_i - \sum_{j=1}^{\beta+1} \lambda_{i, m-j+1}^{-1} \quad (31)$$

where we neglect the superscript of $\mu_i^{(\beta+1)}$ for simplification.

Let $\hat{\alpha}_i$, $i = 1, \dots, N$, denote the number of non-zero eigenvalues of the optimal \mathbf{Q}_i , $i = 1, \dots, N$. $\hat{\mu}_i$, $i = 1, \dots, N$, denote the water-filling level corresponding to the optimal \mathbf{Q}_i , $i = 1, \dots, N$. Then $\hat{\mu}_i$ should satisfy (19) and (20).

We partition $\Phi = \{1, \dots, N\}$ into two subsets, $\Phi_1 = \{i_1\}$, $\Phi_2 = \{i_2\}$. For any element $i_1 \in \Phi_1$, the optimal \mathbf{Q}_{i_1} has $\hat{\alpha}_{i_1}$ non-zero eigenvalues and $\hat{\alpha}_{i_1} \geq \beta + 1$. For any element $i_2 \in \Phi_2$, the optimal \mathbf{Q}_{i_2} has $\hat{\alpha}_{i_2}$ non-zero eigenvalues and $\hat{\alpha}_{i_2} < \beta + 1$.

Suppose Φ_1 is not empty. Then for $i = i_1 \in \Phi_1$, we have $\mu_{i_1} < \lambda_{i_1, m-\beta}^{-1} \leq \lambda_{i_1, m-\hat{\alpha}_{i_1}+1}^{-1} < \hat{\mu}_{i_1}$. By comparing (30) with (19), we have $c < \hat{c}$. By comparing (31) with (20), we have $s_{i_1} < \hat{s}_{i_1}$.

Similar to (23),(24), we have

$$\sum_{i_1 \in \Phi_1} s_{i_1} + \sum_{i_2 \in \Phi_2} s_{i_2} = P, \quad \sum_{i_1 \in \Phi_1} \hat{s}_{i_1} + \sum_{i_2 \in \Phi_2} \hat{s}_{i_2} = P \quad (32)$$

For $i = i_2 \in \Phi_2$, from (32), we know that $s_{i_2} > \hat{s}_{i_2}$. However, by the similar technique in the proof of Lemma 3, we can prove that $s_{i_2} > \hat{s}_{i_2}$ cannot stand. So the assumption that Φ_1 is not empty cannot stand. Therefore, we know the optimal \mathbf{Q}_i has no more than β non-zero eigenvalues, i.e., $\hat{\alpha}_i \leq \beta$.

Now we prove $\hat{\alpha}_i \leq \beta_i$. From the above proof, we know that $c > \hat{c}$. If the optimal \mathbf{Q}_i has β non-zero eigenvalues, then $\mu_i^{(\beta)} > \hat{\mu}_i^{(\beta)} > \lambda_{i, m-\beta+1}^{-1}$. So if $\mu_i^{(\beta)} < \lambda_{i, m-\beta+1}^{-1}$, the optimal \mathbf{Q}_i has no more than $\beta - 1$ non-zero eigenvalues, i.e., the optimal \mathbf{Q}_i has no more than β_i non-zero eigenvalues. \square

Note that, by Algorithm 3, we derive α_i and β_i , the lower bound and upper bound of the rank of the optimal \mathbf{Q}_i respectively, i.e., $\alpha_i \leq \text{rank}(\mathbf{Q}_i^{\text{opt}}) \leq \beta_i$. Also the complexity of Algorithm 3 is very low, since we only use Theorem 1 for N times in Step 1 and do some simple comparison in other steps. We can use Algorithm 2 to find n_i within the lower bound and upper bound derived by Algorithm 3. Therefore, the calculation complexity of Algorithm 2 can be further reduced. Actually, our simulation shows that with Algorithm 3, the number of trials in Algorithm 2 is reduced from MN times to less than $\frac{MN}{2}$ in most cases.

Using Algorithms 2 and 3, we can derive the best power allocation P_i for the MIMO multi-hop channel with reduced complexity compared with Algorithm 1.

IV. RESOURCE ALLOCATION WITH ADAPTIVE TIME DIVISION

In this section, we adopt the adaptive time division strategy, i.e., the time slots T_i allocated to the transmit node and all the intermediate nodes change with the channel conditions to realize the maximum throughput.

By constraint conditions $\sum_{i=1}^N T_i = T$ and (3), we have

$$\sum_{i=1}^N T_i = \sum_{i=1}^N \frac{C}{WC_i} = T. \quad (33)$$

Further, we have

$$C = \frac{WT}{\sum_{i=1}^N \frac{1}{C_i}} \quad \text{bits}. \quad (34)$$

So maximizing C is equivalent to minimizing $f = \sum_{i=1}^N \frac{1}{C_i}$.

From Section III, we know that if n_i denotes the rank of \mathbf{Q}_i , $i = 1, \dots, N$, we have $C_i = \log \prod_{j=1}^{n_i} \mu_i \lambda_{i, m-j+1}$ and $P_i = n_i \mu_i - \sum_{j=1}^{n_i} \lambda_{i, m-j+1}^{-1}$. So the optimization problem can be written as

$$\begin{aligned}
 \min_{\mu_i} f &= \sum_{i=1}^N \frac{1}{\log \prod_{j=1}^{n_i} \mu_i \lambda_{i, m-j+1}} \\
 \sum_{i=1}^N (n_i \mu_i - \sum_{j=1}^{n_i} \lambda_{i, m-j+1}^{-1}) &= P. \quad (35)
 \end{aligned}$$

Once the best μ_i are derived, the best power allocation P_i will be obtained, and further the best time division T_i can be

calculated. The Lagrangian for the above optimization problem is

$$L = \sum_{i=1}^N \frac{1}{\log \prod_{j=1}^{n_i} \mu_i \lambda_{i,m-j+1}} + \rho \left(\sum_{i=1}^N (n_i \mu_i - \sum_{j=1}^{n_i} \lambda_{i,m-j+1}^{-1}) - P \right) \quad (36)$$

where ρ is the Lagrange multiplier.

To minimize L , we take its derivative with respect to all the μ_i and ρ . Then we have the following equations.

$$\frac{n_i \mu_i^{n_i-1} \prod_{j=1}^{n_i} \lambda_{i,m-j+1}}{\prod_{j=1}^{n_i} \mu_i \lambda_{i,m-j+1} \log^2 \prod_{j=1}^{n_i} \mu_i \lambda_{i,m-j+1}} - n_i \rho = 0 \quad i = 1, \dots, N \quad (37)$$

$$\sum_{i=1}^N (n_i \mu_i - \sum_{j=1}^{n_i} \lambda_{i,m-j+1}^{-1}) - P = 0 \quad (38)$$

i.e.,

$$\mu_i \log^2 \prod_{j=1}^{n_i} \mu_i \lambda_{i,m-j+1} = \frac{1}{\rho}, \quad i = 1, \dots, N \quad (39)$$

$$\sum_{i=1}^N (n_i \mu_i - \sum_{j=1}^{n_i} \lambda_{i,m-j+1}^{-1}) = P \quad (40)$$

which can be solved by numerical algorithms easily. Once again, we need to determine the rank of the optimal \mathbf{Q}_i before we can calculate \mathbf{Q}_i . Note that Equations (39)(40) are different from Equations (10)(11) and thus we cannot derive the bounds of the rank of the optimal \mathbf{Q}_i by the similar technique in Algorithm 3. However, we can still find the rank of the optimal \mathbf{Q}_i by a technique similar to the method in Algorithm 2. Once μ_i is derived, we can calculate the best power allocation P_i and the best time division T_i .

V. NUMERICAL RESULTS AND SIMULATIONS

In this section, we provide some numerical results. We assume the total sum power constraint is from 1 W to 100 W. The noise power between any two nodes is 1 W. The total bandwidth is 1 Hz. The total time is N s.

Figure 2 illustrates the average throughput results under four different scenarios. We assume there are four nodes in the multi-hop channel and each node has four antennas. We can see that the throughput derived by Algorithms 2 and 3 is optimal and larger than any other scheme. If the total power budget is equally allocated to each node, some throughput loss will occur compared with the optimal one. When SNR is small, one-dimensional beamforming achieves a throughput that is very close to the optimal one, thus could serve as a simplified and efficient transmission scheme. However, when SNR is large, equal power transmission among all antennas performs better than one-dimensional beamforming.

In Figure 3, similar results can be observed when adaptive time division is adopted. But different from the fixed time division, when the power is equally allocated to each node, the throughput is almost the same as the optimal one. This is because of the fact that the system can adjust time allocation to

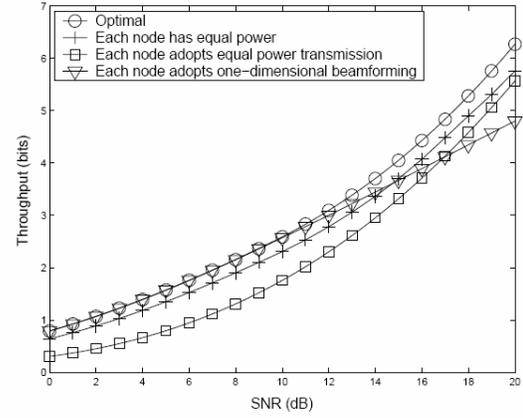


Fig. 2. Throughput comparison of different schemes with fixed time division.

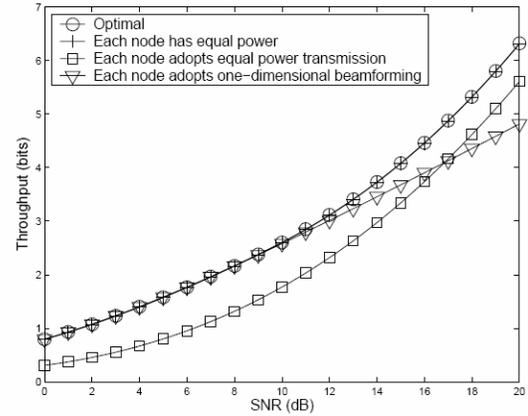


Fig. 3. Throughput comparison of different schemes with adaptive time division.

achieve the maximum throughput. In addition, by comparing the two figures, we can see that the throughput performance with equal time division is very close to the throughput using adaptive time division. Therefore, equal time division can be adopted with high throughput performance and low design complexity.

VI. CONCLUSIONS

In this paper, we investigated the resource optimization in MIMO multi-hop wireless networks. We have reduced the complexity to determine the optimal power and time allocation when either fixed time division or adaptive time division scheme is used. We achieve this by proposing an algorithm in which we determine the power allocation and the rank of the optimal transmit covariance matrix separately instead of jointly. In addition, we add a pre-processing algorithm to reduce the complexity further. We find that while dynamical allocation of time and power could increase channel capacity, equal time and power allocation among different nodes may not cause much capacity loss. Therefore, we can obtain a trade-off between the capacity performance and system complexity by choosing different resource allocation schemes.

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